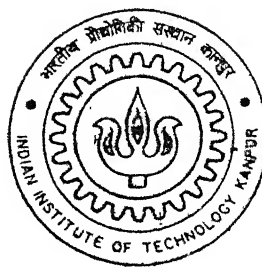


# **VIBRATION CONTROL OF COMPOSITE PLATE BY USING PIEZOELECTRIC CRYSTALS**

By

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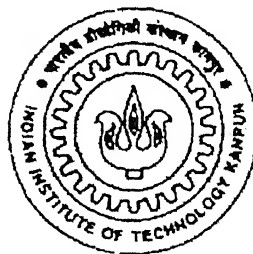
**MARCH, 2002**

# VIBRATION CONTROL OF COMPOSITE PLATE BY USING PIEZOELECTRIC CRYSTALS

*A Thesis Submitted  
In Partial Fulfillment of the Requirements  
for the Degree of  
Master of Technology*

*by*

**SAHEBRAO MUNESHWAR**



to the  
**DEPARTMENT OF AEROSPACE ENGINEERING  
INDIAN INSTITUTE OF TECHNOLOGY KANPUR**

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सर्वोत्तम काशीबाग के बाहर पत्रकालिका  
भारतीय प्रौद्योगिकी संस्थान, मुंबई  
अवधि क्र० A. 139560



# CERTIFICATE

18/03/2002  
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It is certified that the work contained in this thesis entitled "VIBRATION CONTROL OF COMPOSITE PLATE BY USING PIEZOELECTRIC CRYSTALS" by Sahebrao K . Muneshwar , has been carried out under my supervision and this work has not been submitted elsewhere for a degree.

March 2002,

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# ABSTRACT

The investigation in this thesis deals with the vibration control of first three modes of a composite plate embedded with actuators and sensors by using feedback control system with proportional control action. The plate was excited by electrodynamic vibrator at three different frequencies and sensor responses were being taken by using FFT analyzer and PC. Analog output from PC was amplified and stepped-up through amplifier & transformer respectively. This voltage was applied to a pair of crystals to control the vibration of the plate at that particular forcing frequency. Different forcing amplitude to control the vibration of the clamped-clamped plate were tried but it was found that the exciting force required for vibration control was too high as compared to the force generated by the actuators at maximum voltage. So the same plate with cantilever boundary condition was used in vibration control studies.

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# Chapter 1

## INTRODUCTION AND LITERATURE REVIEW

### 1.1 General

Investigation of dynamics of smart materials has been drawing attention of many researchers worldwide during the past 10 years or so. A smart structure has embedded actuators and sensors and an active electronic controller that modifies dynamical behavior of the structure. The smart structure can help to reduce the amplitude of vibration, bring down the overall mass, alleviate large concentration of loads at critical parts, increase fatigue life, reduce the drag on the wing and noise inside a fuselage. They can be used on all Aerospace vehicles without exception. The application of smart structure in aerospace industry is limited only by imagination during the evolution of technology. The present work is a small step in this direction.

The word composite means, "consisting of two or more distinct phases". Thus materials having two or more distinct constituents, on a macro scale having distinct interface separating them are called composite materials. The composites achieve certain physical properties not realizable by the constituent materials individually. These materials offer high specific strength and stiffness. Oriented fibrous composites also offer controlled anisotropy. The manufacture of composites requires comparatively low labour and generates less wastage in addition to the ease of processing of complicated structural forms. Composite structures are destined to many future applications, especially in Aerospace and transport industry due to their properties such as good fatigue and corrosion resistance, low heat conductivity, good electrical insulation properties & favourable cost effectiveness, etc. Also in the recent past extensive research on the manufacture, characterization, fatigue and fracture of these materials have increased the confidence in the use of these materials and thus they have replaced conventional materials in many applications.

With progress in engineering design, increasing use, is being made

of lightweight high strength materials. This is true irrespective of the field of their application whether could be buildings, bridges, aerospace structures or mechanical structures. These structures are lighter, more flexible and are made of materials that provide much lower energy dissipation and it may contribute to more intense vibration response. As we know any external disturbance will cause deformations and stresses in the structures and these minor changes in shape may affect the performance adversely and may result in the malfunctioning of the whole system.

## 1.2 Smart materials

The synergistic interaction of this field has led to the birth of the new trans disciplinary field of Smart/Intelligent Structures Technology. 'Smart', 'Intelligent' and 'Adaptive' have all been used to describe and classify structures which contains sensors, actuators and control capabilities. The "smartness" or "intelligence" can be defined at three levels, each of them defining a field of research, technology development and potential applications:

1. A material or a structure is said to be sensitive when it includes sensors providing information concerning the material itself or its environment.  
For instance the control of large and complex technological structures is a topic of rapidly growing interest.
2. A material will be adaptable if integrated actuators (or active materials) can modify its characteristics. Such a material or structure will be adaptable only by the way of an externally determined action.
3. The combination of these two above mentioned properties results in an "adaptive" or "really smart" material which collects data related to the changes in its environment or in its own damage. It processes these collected data and reacts through its 'actuators' action.

The design of smart structure is significantly dependent on the appropriate selection of smart materials. The available smart materials can be classified into five major groups based on the characteristic working principles: Piezoelectric, Magnetostrictive, Shape memory alloys, Electro-rheological and magneto-rheological materials and optical fibers. Piezoelectric materials develop charge if deformed by the mechanical stress and by a converse effect, it deforms due to the application of an electric field. A similar

direct and converse relationship between mechanical and magnetic field exist for magnetostrictive material. The majority of the other types of materials are however known to possess only unidirectional effects. Shape memory alloys, for example, have special features of memorizing a shape stretching, bending or twisting as a function of temperature and recovering that shape at different temperatures. They deform due to phase change from Martensite to Austenite state. The phase transition is dependent on both stress and temperature such that by changing any one of them, volume change can be initiated. The electro-rheological fluids are a class of specially formulated colloidal suspensions, which undergo a change in the resistance to flow due to applied electric field.

The last two materials can be very effectively used for vibration suppression by embedding actuators out of them while the piezoelectric and magnetostrictive materials are being used for both sensing and actuation by making use of direct and converse relationship existing in such materials. Each smart material possesses certain unique properties, which makes it attractive for a specific application. Shape memory alloy for example is suitable for static shape control whereas Magnetostrictive and piezoelectric materials are more useful in dynamic applications. However, piezoelectric crystals and magnetostrictive Terfenol-D rods are more attractive due to their good operational bandwidth and moderate capacity of generating actuation strain. Polyvinylidene fluoride (PVDF), yet another type of piezoelectric material, is relatively inferior as actuator since it has poor structural strength and the strain induction is also quite small. However, PVDF can be used as a very good sensor, as it interferes minimally with the host structural responses.

Presently, piezoelectric and magnetostrictive materials are more readily used as smart elements due to the reliability of their physical properties and dual applicability as sensor and actuator. This thesis presents the study of piezoelectric crystals in vibration suppression.

## 1.3 Piezoelectric materials

Piezoelectricity, in other word means 'Electricity from Pressure'. The so called 'direct effect' generates electric charge on application of mechanical stress over piezoelectric crystals. The inverse effect in piezoelectricity is production of the deformation in crystals on application of the voltage. Strong piezoelectric effect has also been discovered in a polymeric crystals called Polyvinylidene fluoride (PVDF). They can be employed both as sensors and actuators in the development of smart structures.

Piezoelectric ceramic materials are typically employed as sensors and actuators, while piezoelectric polymers are typically employed as sensors. Recently, electrostrictive materials are being tipped to replace well known PZT based actuator when the cost of the former gets reduced due to its increased acceptance and usage.

## 1.4 Sensors and actuators

Many types of sensors and actuators are being considered for introduction in smart systems. Typically sensors are being used as strain gauges, accelerometers, fiber optics, piezoelectric films and piezoceramics. Sensors convert strain or displacement (or their time derivatives) into electric field. Key factors for sensors are their sensitivity to strain or displacement, bandwidth and size. Other factors are temperature sensitivity, linearity, hysteresis, electromagnetic compatibility and embeddability.

Typical actuators consist of piezoceramics, magnetostrictives, electrostrictives and shape memory alloys. Piezoelectrics and electrostrictives are available as ceramics, whereas magnetostrictives and shape memory alloys are available as metal alloys. Most important performance parameters of actuators include maximum strain, maximum block force, stiffness and bandwidth. All these actuators directly convert actuating (electric/thermal) signals into actuation strain/displacement.

At the present time, the primary application being investigated is vibration control. There has been some significant work investigating shape control and damage control of structures but not nearly to the extent of active and passive vibration control. Passive techniques utilize prior adjustment of the three design parameters mass, stiffness and damping so that the structure gives a good dynamic response to an outside disturbance. These methods include tuning masses to place natural frequencies away from driving frequencies, vibration absorbers which create anti-resonance and additional damping. Accepted methods are limited, however, to a specified range of operating conditions. Active control techniques allow for a much larger range of operating conditions. Present active control systems require the use of applied external forces or motion at discrete locations to oppose the vibration and effectively dissipate the energy in the structure. A structure with the ability to change its stiffness or damping in a controlled manner would offer an alternative to discrete external actuators. Smart structure applications are wide ranging from active shape control, vibration and noise control, improved damping, improved aeroelastic stability.

# Literature review

Investigation on dynamics of smart structures have been drawing attention of many researchers world wide during the past ten years or so. The field of smart materials and structures is emerging rapidly due to technological innovations in engineering materials , sensors , actuators and image processing. Smartness is defined by Gandhi and Thomson [1], Rogers[2] and Frank[3] as follows: Smartness describes self adaptability , self sensing ,memory and multiple functionality of materials or structures. These characteristics provide numerous possible applications for smart materials and structures in Aerospace , manufacturing , civil infrastructure systems and biomechanics.

World wide , many researchers have paid their attention towards the development of intelligent structures and have shown encouraging results in this field. The problem of exerting control on large or precision flexible structure has, in recent years, been the center of attraction of an exhaustive research effort throughout the Aerospace engineering field. Many presently planned and future missions such as structural health monitoring, vibration control, airfoil shape control, noise control, control surface actuation, smart radar, etc. need for pointing accuracy for better performance execution. World wide, many researchers have paid their attention towards the development of intelligent structures and have shown encouraging results in this field.

A detailed literature survey of the issues related to properties of PZT ceramics and their use as actuators for vibration control is not attempted in this chapter because of the extensive activity in this field. However, typical references are mentioned which provide some measure of background for the various issues of concern. The actuators in a smart structure provide the mechanism for the structure to adapt to its surrounding by suppressing vibrations or changing the structure's shape. This requires an actuation authority over a broad bandwidth spectrum and over a wide range of displacement amplitudes.

Hagood et al [4] have shown that piezoelectric crystals can readily be embedded in composite structures without much affecting the global stiffness of the substructure and increasing the mass of the total system, to maximize the energy dissipation in the structure and exhibiting control on the vibrations. Another feature of their work is the development of suitable technique of embedding piezoelectric crystals in a composite laminate. They have shown that piezoelectrics are uniquely suited as elements of highly distributed sensors and actuators in smart structures.

Crawley and Luis et al [5] have investigated the use of piezoelectric



actuators to excite steady-state resonant vibrations in simple structural members such as cantilever beams. The response of the specimens were found in good agreement with those predicted by the analytical models.

In order to study the possibility of detecting and monitoring damage in composite laminates through piezoelectric film sensors, three different laminates in the presence of various damage were investigated and influence of damage on monitoring sensors was specifically analyzed.

Zhou et al [6] have presented a dynamic analytical approach for the design and integration of active piezoceramic (PZT) patch elements locally coupled with host structures. Several critical design issues have been addressed like determination of the actuator dynamic outputs, the prediction of energy conversion efficiency, the estimation of system power requirement and the limitation of induced alternate peak stress. Both the mechanical stress behavior and the thermal stress characteristics of the PZT patch elements were investigated. The attention in parametric design was directed to the thickness and location of the elements.

Batra et al [7] have shown the determination of the optimum location of a given rectangular piezoelectric actuator that will require the minimum voltage to annul the deflection of a simply supported rectangular elastic plate vibrating near one of its fundamental frequencies.

Lee [8] presented theory of laminated piezoelectric plates for the design of distributed sensors/actuators. In his theory, the piezoelectric phenomenon to effect distributed control and sensing of bending, torsion, shearing, shrinking and stretching of a flexible plate has been developed. The theory is capable of modeling the electromechanical (actuating) and mechano-electrical (sensing) behavior of a laminate.

Akella et al [9] have shown the modeling and control issues related to smart structures bonded with piezoelectric sensors and actuators. They applied Hamilton's principle to obtain a linearized equation of motion. The natural modes are then found by solving an eigenvalue problem. Shen [10] developed a one-dimensional theory for modeling the analysis of beams containing piezoelectric sensors and actuators. The equations of motion and associated boundary conditions are derived for the vibrations of piezoelectrically sensed/actuated beams, plates as well.

## Present work

Investigations on dynamics of smart structures have been drawing attention of many researchers worldwide. Smart materials, may be defined as materials that possess the adaptive capabilities to external stimuli such as load or environment with inherent intelligence. The intelligence of the material could perhaps be programmed by material composition, processing, defect and microstructure or conditioning to adapt in a controlled manner to various levels of stimulus. Piezoelectric ceramic materials are typically employed as sensors and actuators.

Although significant advances have been made elsewhere to control the vibration of composite structure embedded with PZT crystals by using feedback control system, the field is still in its infancy in India. A number of groups (such as those at IISc Bangalore and NAL, Bangalore) have been working on theoretical aspects relating to smart structure applications for the last few years. There is a need to develop the expertise on experimental aspects relating to smart structure applications within our country.

Accordingly the objectives of the current endeavors are two fold:

- (1) To study the vibration responses of composite plate embedded with these crystals at various boundary conditions.
- (2) To control the vibration of composite structure by using closed loop control system.

Various processing stages and experimental techniques employed are described in chapter 2 & 3. Theoretical analysis of the plate vibration response is described in chapter 4. Chapter 5 encompasses results and discussion. Chapter 6 deals with conclusions and suggestions for future work.

# Chapter 2

## ELEMENTS OF FEEDBACK CONTROL SYSTEM

### 2.1 Feedback control system

The basic elements of a feedback control system shown in figure-1. Block diagram represents the flowpaths of control signals. The control elements are components needed to generate the appropriate control signal applied to the plant. These elements are also called “controller”. In other way, the feedback elements are components needed to identify the functional relationship between the feedback signal and the controlled output. The feedback signal is a function of the output signal. It is sent to the summing point and added to the reference input signal to obtain the actuating signal. The actuating signal represents the control action of the control loop and is equal to the sum of the reference input signal and feedback signal. This is also called as “error signal”.

An automatic controller compares the actual value of the plant output with the desired value, determines the deviation and produces a control signal which will reduce the deviation to zero or to a small value. The manner in which the automatic controller produces the control signal is called the control action. Controllers classified according to their control action:

1. two-position or on-off controllers
2. proportional controllers
3. integral controllers
4. proportional-plus-integral controllers
5. proportional-plus-derivative controllers
6. proportional-plus-derivative-plus-integral controllers

The current work deals with the feedback control system with proportional control action. For a controller with proportional controller action, the relationship between

the output of the controller  $u(t)$  and the actuating error signal  $e(t)$  is:

$$u(t) = k_p e(t)$$

or in Laplace transformed equations,

$$\frac{U(s)}{E(s)} = k_p$$

Where,

$k_p$  is the proportional gain.

Whatever the actual mechanism may be and whatever the form of the operating power, the proportional controller is essentially an amplifier with adjustable gain. A block diagram of proportional control action is shown in figure-2.

### 2.1.1 PCL-812 card

The PCL-812 is high performance, high speed, multi-function data acquisition card for IBM PC/XT/AT and compatible computers. The high-end specifications of this full-sized card and complete software support from third party vendors make it ideal for a wide range of applications in industrial and laboratory environments. These applications include data acquisition, process control, automatic testing and factory automation.

This was used to process the signals coming from the sensors for analyzing the smart structure responses and getting a output voltage in the range of  $\pm 1 V$ . In addition to it other electrical units are needed to have the required output voltage such as RC circuit to convert DC power to AC power, Amplifier and Transformer to amplify the same.

#### Key features

1.16 single ended analog input channels.

2. Switch selectable versatile analog input ranges. Bipolar:  $\pm 1V, \pm 2V, \pm 5V, \pm 10V$ .

3. The ability to transfer A/D converted data by program control, interrupt handler routine or DMA transfer.

4. An industrial standard 12-bit successive approximation converter to convert analog inputs. The maximum A/D sampling rate is 30KHz in DMA mode.

### 2.1.2 RC circuit

It is a simple circuit to convert the DC power to AC power. It consists of wirewound resistance, feed capacitor and variable resistance. The main function of the wirewound is to resist the DC component and allow AC components to pass through it. But it is not a fully efficient component, as the output contains some DC components also. So to bypass this DC components, a variable resistance was connected in parallel with the wirewound and feed capacitor. By adjusting the variable resistance the AC output purity could be improved.

### 2.1.3 Inverter

It makes the input signal out of phase by  $180^\circ$ . To have the signal of  $180^\circ$  out of phase with the input signal this circuit was introduced in the feedback system after the RC circuit. It was also acting as a buffer to protect the PC from high voltage.

### 2.1.4 PU-amplifier & transformer

We were having output of max 1V/10 mA from the PC. It was simply not sufficient to control the vibration of the plate, needed to amplify this power level. We selected the 'C' class audio amplifier to amplify the voltage as well as current. The amplifier may be less than ideal in that the output voltage or current may not be independent of the load.

The amplification factor of this amplifier is too high, it is about 100. With the help of this amplifier we were able to amplify the power up to 52 V. There is one major disadvantage of this amplifier which is that it amplifies the noise also. To operate this effectively, we have used a filter which to reduce the noise level and allow only signal to amplify.

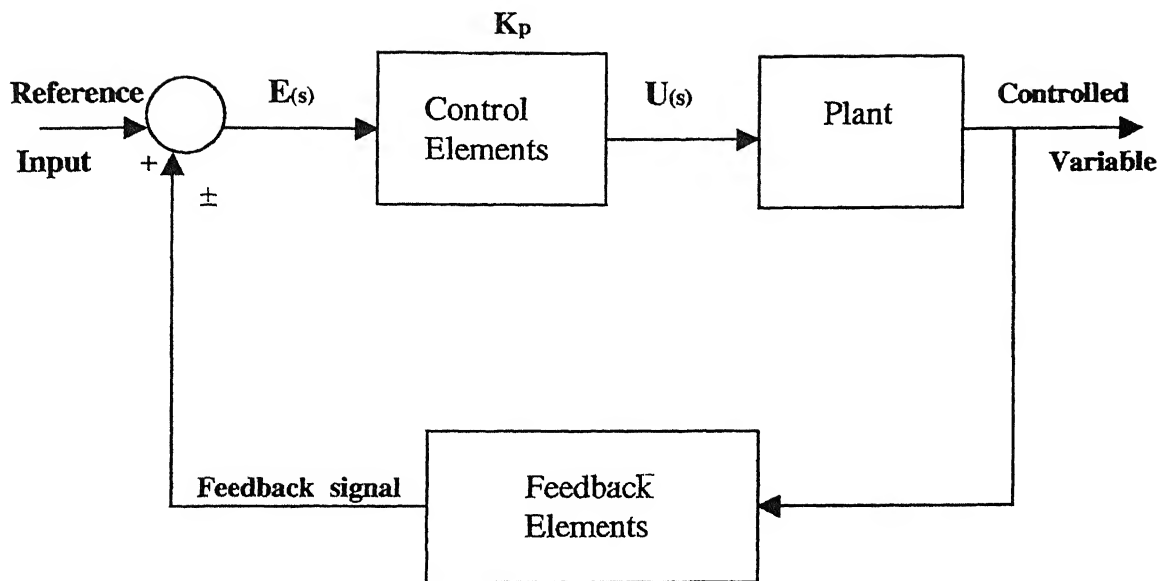


Figure-1:Block diagram of feedback control system

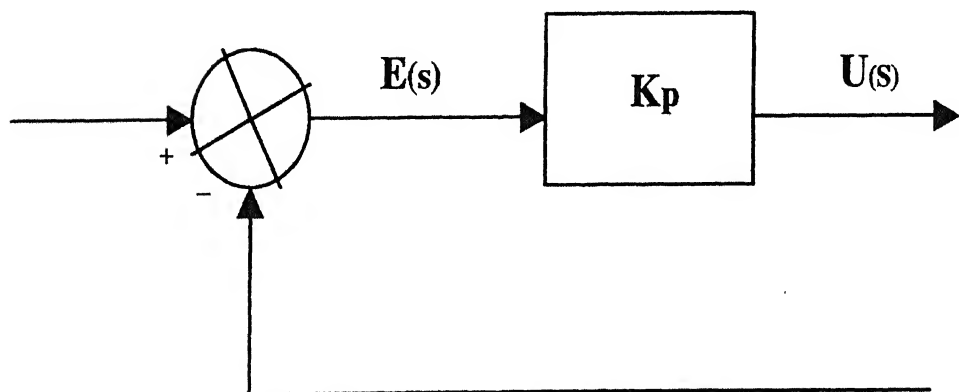


Figure-2:Block diagram of proportional controller

# Chapter 3

## CHARACTERIZATION OF COMPOSITE PLATE

### 3.1 Composite laminate

The most prominent feature of the reinforced polymeric composite is its high strength to weight ratio as compared to the bulk materials. Also compared with the strength of bulk materials, man made fibers of nonpolymeric materials exhibit much higher strength along their length as large flaws, which may be present in the bulk materials are minimized because of the small cross-sectional dimensions of the fibers. A high fiber aspect ratio (length to diameter ratio) permits very effective transfer of load through matrix materials to the fibers. Glass fibers are the most common of all reinforced fibers for polymeric composites due to their low cost and high strength.

#### 3.1.1 Basic raw materials

The specimen used in the present work for all the test was glass fabric/epoxy composite. The basic raw materials for the preparation of the laminate were the glass fabric (reinforced material) and epoxy mixture (matrix material). The specifications of these materials are given in the Table-1 and 2.

##### a. Glass woven fabric

Woven fabrics are of the most widely used reinforced materials. Woven reinforcing fabrics are made by interlacing individual filaments, ends (untwisted fiber bundles), yarns (twisted fiber bundles) and rovings. Fabric composites have mechanical properties similar to those of laminates made from orthogonal unidirectional layers. Fibers can be woven into different types of weave patterns, widths and thicknesses. A variety of weave patterns can be used to interlace warp and weft (filling) yarns to form a stable

fabric. The weave pattern controls the handling characteristics of a fabric and to some degree, the properties of the product that uses it as reinforcement. The major styles are plain weaves, twills, satins and woven rovings. The plain weave interfaces one warp yarn over and under the weft (or filling) yarn. Yarn count and content, however, also contribute to fabric stability.

#### **b. Epoxy resin system**

One of the most versatile property of resins is their ability to transform readily from the liquid state to tough, hard thermoset solids. Hardening is accomplished by the addition of a chemically active reagent known as curing agent or hardener. Accelerators are sometimes added to reduce the curing time. The specific resin curing agent will determine the ultimate properties of the laminate to a considerable extent. The composition of the epoxy resin employed is given in Table-2.

### **3.2 Preparation of the laminate**

The glass fabric reinforces the epoxy resin matrix. The glass fiber layers were cut from the glass fiber cloth of required size. The three layers were cut, of crystal size at the center at equal distance from the neutral axis to place the crystals. We prepared laminate by using the mixture of Araldite and Hardener as epoxy. First the Hardener and Araldite were being taken separately in the ratio of 1:10. Araldite was heated upto 110 °C to eliminate the air-bubbles and then allowed to cool upto room temperature. Then Araldite, Hardener mixture ready to use for preparing the laminate. Prepregs can be stacked together to get the laminate of required size. This laminate was then cured in a hydraulic press. The laminate was placed between the two plates of the press. Then the pressure is gradually raised to 0.5 Mpa in half an hour. Now this pressure is maintained for 24 hours and it was then allowed to cool to room temperature. The cured laminate was taken out of the press and cut to required sized specimens by a diamond cutter.

For composite, there are four independent elastic constants; the elastic moduli in the longitudinal and transverse directions, the shear modulus and the major Poisson's ratio. Various tests were performed to characterize the properties of the composite laminate.

#### **Specimen preparation**

The geometry of the glass fabric/epoxy specimens for the tensile tests are shown in figure-3 as per ASTM Standard D3878. The distance between the tabs (gauge



length) is 130 mm and the specimens width is about 35 mm. Specimens are cut from the composite plate of layers prepared by curing. The end tabs length ranges from 35 to 40 mm and the tab thickness is 2 mm are cut from the aluminium strip of length 380 mm as shown in figure-4. Specimens were cut using a diamond cutter from plates and then bonded with the aluminium tabs. 12 mm long strain gauges were bonded using Araldite and Hardener on the specimens along loading and perpendicular to loading directions. Strain gauges were covered with wax to prevent from absorbing moisture.

### Tensile test

Tests were carried out on various specimens cut along  $0^\circ$ ,  $90^\circ$  and  $\pm 45^\circ$  fiber orientations (warp direction) of the composite plate for determining  $E_T$ ,  $E_L$  and  $G_{LT}$  respectively.

### 3.3 Test procedure

For characterizing the material, tension tests were performed. All the tests were carried out on MTS - 810 machine. The tensile test was performed utilizing the wedge-section friction grips. The specimen was first aligned in the grips and tightened in place. The specimen was loaded up to failure at a rate 2.00 mm/min. Strains were measured by the strain gauges connected to a strain indicator. The load Vs strain curve for various specimens are shown in figure-5. The data was analyzed to estimate elastic moduli and inplane shear modulus.

Burn out test was carried out to calculate the exact fiber volume fraction in the laminate and finding out the various properties of the laminate by applying the rule of mixtures. The significance of this method is it can be used to obtain the ignition loss of a cured reinforced resin sample. If any glass fabric is used as the reinforcement of an organic resin that is completely decomposed to volatile materials under the conditions of this test and the small amount of volatiles (water, residue solvent) that may be present is ignored, the ignition loss can be considered to the resin content of the samples.

This method does not provide a measure for resin content of the samples containing reinforce materials that loose weight under the conditions of the test or containing resin that does not decompose to volatile materials released by ignition. Ceramic crucibles were used. Electric furnace, variac controlled electric furnace capable of maintaining  $1350^\circ\text{C}$  was used for this test.

For the sake of homogeneity samples were cut from the specimens that had been used for mechanical properties determination. The fracture areas were removed

and three specimens of the sizes given in Table-3, were prepared.

The crucibles were heated to 500 to 600 °C for 15 minutes or more, cooled to room temperature in a dessicator and weighed. The composite specimens were placed in the respective crucibles and weighed accurately. Weight of the different crucibles, the specimen which were placed in the crucibles and the weight of the crucibles including specimens before and after test are summarized in the following Table-4.

They were held at 600°C temperature for 6 hours, the time until all carbonaceous material has disappeared. They were cooled to the room temperature in the dessicator and weighed. Since weight of the fiber and matrix and the density of both the fiber and matrix are known, one can easily calculate the volume fraction of the fiber. The volume fraction of the fiber was found to be 0.46 on average. The difference in fiber fraction of the various samples were within 0.8%. The density of the laminate was 1.78 gm/cm<sup>3</sup>.

Material	Glass Fabric
Weave	Plain weave
No. of Yarns per inch	Warp:40 Weft:36
No. of fiber per Yarn	Warp:34 × 2 Weft:34
Diameter	9 Mcrons
% Fiber	Warp:69 Weft:31

**Table-1: Specifications of glass fabric**

Sr.No	Composition	Specification	Parts by weight
1	Araldite	CY230	100
2	Hardener	HY951	9-10
3	Accelerator	HY73	1

**Table-2: Composition and specification of epoxy resin**

Specimen Number	Size of the specimen (cm×cm)
1	2.0×2.15
2	2.05×2.15
3	2.0×2.20

**Table-3:Burn out test specimen sizes**

Sr.No	Specimen weight (gm)	Crucible's weight	Crucible's weight with specimen (before burnout)	Crucible's weight with specimen (after burnout)
1	2.1459	45.9964	48.1423	47.2033
2	2.2437	37.3942	39.6379	38.6390
3	2.3885	41.8910	44.1795	43.1620

**Table-4:Weights of the glass fabric/epoxy specimens before and after the burnout test**

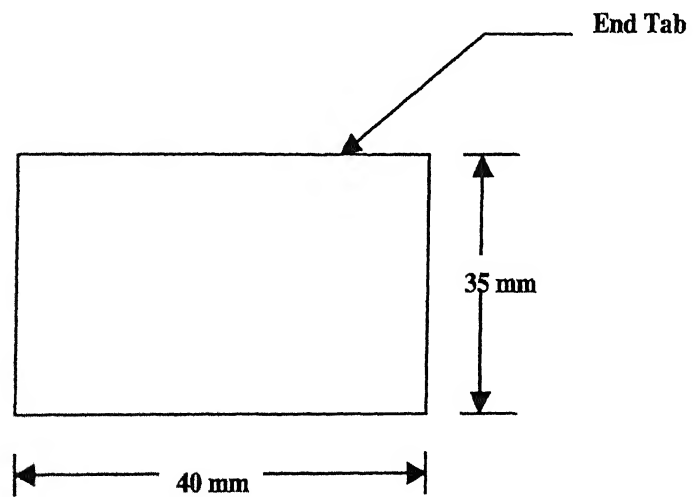
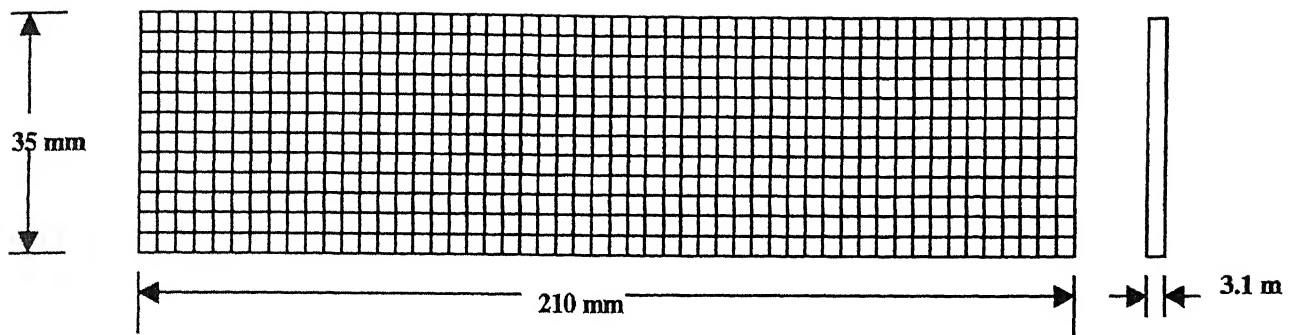
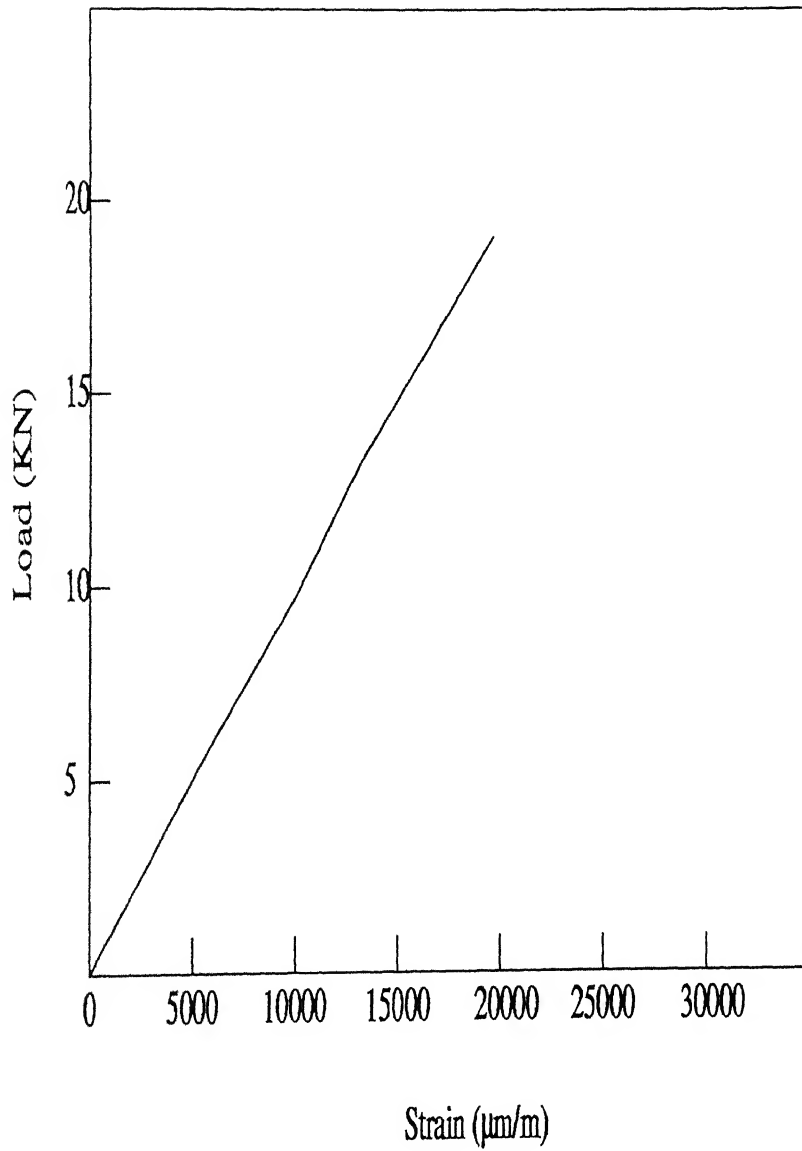


Figure-3&4: Tensile test specimen with  $0^{\circ} - 90^{\circ}$  fiber orientation & end tab



**Figure-5:Load Vs Strain curve of (0-90 degree orientation) glass fabric/epoxy**

# Chapter 4

## PLATE VIBRATION RESPONSE

### 4.1 Theoretical formulation of the plate vibration response

The objective of this analysis is to predict natural frequencies and the deflection of the plate embedded with the piezoelectric sensors and actuators as well. The Plate was clamped at two edges and free at the other is as shown in figure-6 , considered for the analysis.

For deriving the governing equations following assumptions are being made:

- 1.The plate is thin,i.e. the thickness of the plate is much smaller than the other physical dimensions.
2. Transverse shear stresses in  $\tau_{xz}$  and  $\tau_{yz}$  are negligible.
- 3.The transverse normal stress  $\sigma_z$  is negligible.
- 4.Inplane displacements  $u$  and  $v$  are linear functions of  $z$ -coordinate.
- 5.Material behavior is linearly elastic.
- 6.The masses of the sensors and actuators are negligible compared to the mass of the plate.
- 7.Structural damping is neglected.

It should be noted that assumptions 2 and 4 constitute the classical assumptions of Kirchhoff. A specially orthotropic plate configuration is considered. Therefore the laminate stiffness consists only  $D_{11}$  ,  $D_{12}$  ,  $D_{22}$  and  $D_{66}$  elements.

Any continuous structure is a system with infinite degrees of freedom but to avoid complications in mathematical modeling , it can be considered as a system with  $N$  degrees of freedom.

The lateral deflection  $W(x,y,t)$  of any system can be approximated as

$$W(x, y, t) = \sum_{i=0}^n \sum_{j=0}^n q_{ij}(t) \Phi_{i(x)} \Psi_{j(y)} \dots \dots \dots (1)$$

Where,

$q_{ij}$  are the generalized coordinates.

$\Phi_i$  and  $\Psi_j$  are assumed dimensionless shape functions which satisfy the prescribed boundary conditions.

$$\Phi_{i(x)} = \{ \sin(\beta x) - \sinh(\beta x) - \cos(\beta x) + \cosh(\beta x) \}$$

$$\Psi_{j(y)} = \{ \sin(\eta y) + \sinh(\eta y) - \cos(\eta y) - \cosh(\eta y) \}$$

Lagrange's equations of motion are

$$\frac{\partial}{\partial t} \left( \frac{\partial}{\partial \dot{q}_{rs}} \right) - \frac{\partial T}{\partial q_{rs}} + \frac{\partial V}{\partial q_{rs}} = Q_{rs}$$

Reduced to ,

$$\frac{\partial}{\partial t} \left( \frac{\partial T}{\partial \dot{q}_{rs}} \right) + \frac{\partial V}{\partial q_{rs}} = \{0\} \dots \dots \dots (2)$$

Where,

$T$  = Total kinetic energy

$V$  = Total strain energy

$q_{rs} = q_{11}, q_{12}, \dots, q_{1n}, q_{21}, \dots, q_{nn}$  , are the generalized coordinates

Kinetic energy of the plate is given by

$$T = 1/2 \int_0^a \int_0^b m (\dot{W})^2 dx dy \dots \dots \dots (3)$$

Where ,

$a$  and  $b$  are length and breadth of the plate respectively and  $m$  is the mass per unit area of the plate.

Non-dimensionless form ,

$$T = \frac{1}{2} \int_0^1 \int_0^1 m \left( \dot{W} \right)^2 d\zeta d\eta$$

For a specially orthotropic plate (i.e  $D_{16}=D_{26}=0$ ),the strain energy given by

$$V = 1/2 \int_0^a \int_0^b \left\{ D_{11} \left( \frac{\partial^2 w}{\partial x^2} \right)^2 + D_{22} \left( \frac{\partial^2 w}{\partial y^2} \right)^2 + 2D_{12} \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} + D_{66} \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 \right\} dx dy \dots\dots\dots(4)$$

$$V = \frac{1}{2} \int_0^1 \int_0^1 \left\{ D_{11} \left( \frac{\partial^2 w}{\partial \zeta^2} \right)^2 + D_{22} \left( \frac{\partial^2 w}{\partial \eta^2} \right)^2 + 2D_{12} \frac{\partial^2 w}{\partial \zeta^2} \frac{\partial^2 w}{\partial \eta^2} + D_{66} \left( \frac{\partial^2 w}{\partial \zeta \partial \eta} \right)^2 \right\} d\zeta d\eta$$

**Kinetic energy:**

$$T = 1/2 \int_0^a \int_0^b m \left( \dot{W} \right)^2 dx dy$$

$$T = m/2 \int_0^a \int_0^b \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n \sum_{l=1}^n \dot{q}_{ij} \dot{q}_{kl} \Phi_{i(x)} \Psi_{j(y)} \Phi_{k(x)} \Psi_{l(y)} dx dy$$

$$T = \frac{m}{2} \sum_{i=0}^n \sum_{j=0}^n \sum_{k=0}^n \sum_{l=0}^n \dot{q}_{ij} \dot{q}_{kl} \left[ \int_0^a \Phi_{i(x)} \Phi_{k(x)} dx \right] \left[ \int_0^b \Psi_{j(y)} \Psi_{l(y)} dy \right]$$

$$= \frac{m}{2} \sum_{i=0}^n \sum_{j=0}^n \sum_{k=0}^n \sum_{l=0}^n \dot{q}_{ij} \dot{q}_{kl} I_{\Phi_i \Phi_k} I_{\Psi_j \Psi_l} \dots\dots\dots(5)$$

**Potential energy :**

Deflection is given by

$$W(x, y, t) = \sum_{i=1}^n \sum_{j=1}^n q_{ij}(t) \Phi_{i(x)} \Psi_{j(y)}$$

Then

$$\frac{\partial W}{\partial x} = \sum_{i=1}^n \sum_{j=1}^n q_{ij} \Phi'_{i(x)} \Psi_{j(y)}$$



$$\frac{\partial w}{\partial y} = \sum_{i=1}^n \sum_{j=1}^n q_{ij} \Phi_{i(x)} \Psi'_{j(y)}$$

$$\frac{\partial^2 w}{\partial x^2} = \sum_{i=1}^n \sum_{j=1}^n q_{ij} \Phi''_{i(x)} \Psi_{j(y)}$$

$$\frac{\partial^2 w}{\partial y^2} = \sum_{i=1}^n \sum_{j=1}^n q_{ij} \Phi_{i(x)} \Psi''_{j(y)}$$

$$\frac{\partial^2 w}{\partial x \partial y} = \sum_{i=1}^n \sum_{j=1}^n q_{ij} \Phi'_{i(x)} \Psi'_{j(y)}$$

Consider

$$\begin{aligned} \int_0^a \int_0^b \left( \frac{\partial^2 w}{\partial x^2} \right)^2 dx dy &= \int_0^a \int_0^b \sum_{i=0}^n \sum_{j=0}^n \sum_{k=0}^n \sum_{l=0}^n q_{ij} q_{kl} \Phi''_{i(x)} \Psi_{j(y)} \Phi''_{k(x)} \Psi_{l(y)} dx dy \\ &= \sum_{i=0}^n \sum_{j=0}^n \sum_{k=0}^n \sum_{l=0}^n q_{ij} q_{kl} \left[ \int_0^a \Phi''_{i(x)} \Phi''_{k(x)} dx \right] \left[ \int_0^b \Psi_{j(y)} \Psi_{l(y)} dy \right] \\ &= \sum_{i=0}^n \sum_{j=0}^n \sum_{k=0}^n \sum_{l=0}^n q_{ij} q_{kl} I_{\Phi_i'' \Psi_k''} I_{\Psi_j \Psi_l} \dots \dots \dots (6) \end{aligned}$$

Similarly

$$\int_0^a \int_0^b \left( \frac{\partial^2 w}{\partial y^2} \right)^2 dx dy = \sum_{i=0}^n \sum_{j=0}^n \sum_{k=0}^n \sum_{l=0}^n q_{ij} q_{kl} I_{\Phi_i \Phi_k} I_{\Psi_j'' \Psi_l''} \dots \dots \dots (7)$$

$$\int_0^a \int_0^b \left( \frac{\partial^2 w}{\partial x^2} \right) \left( \frac{\partial^2 w}{\partial y^2} \right) dx dy = \sum_{i=0}^n \sum_{j=0}^n \sum_{k=0}^n \sum_{l=0}^n q_{ij} q_{kl} I_{\Phi_i'' \Phi_k''} I_{\Psi_j'' \Psi_l''} \dots \dots \dots (8)$$

$$\int_0^a \int_0^b \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 dx dy = \sum_{i=0}^n \sum_{j=0}^n \sum_{k=0}^n \sum_{l=0}^n q_{ij} q_{kl} I_{\Phi_i' \Phi_k'} I_{\Psi_j' \Psi_l'} \dots \dots \dots (9)$$

Using equations (4),(6),(7),(8) and (9)

We get,

$$V = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n \sum_{l=1}^n \left[ D_{11} I_{\Phi_i'' \Phi_k''} I_{\Psi_j \Psi_l} + 2D_{12} I_{\Phi_i'' \Phi_k''} I_{\Psi_j \Psi_l'} + 4D_{66} I_{\Phi_i' \Phi_k'} I_{\Psi_j \Psi_l'} + D_{22} I_{\Phi_i \Phi_k} I_{\Psi_j'' \Psi_l''} \right] \dots\dots\dots(10)$$

To employ Lagrange's principle , we need to know the virtual work done by the externally applied concentrated load 'F' at the center of the plate.This can be calculated as

$$\begin{aligned} \delta V &= F (\delta W) |_{(a/2, b/2)} \\ &= F \sum_{i=0}^n \sum_{j=0}^n \delta q_{ij} \Phi_{i(a/2, b/2)} \Psi_{j(a/2, b/2)} \\ &= \sum_{i=0}^n \sum_{j=0}^n F \Phi_{i(a/2, b/2)} \Psi_{j(a/2, b/2)} \delta q_{ij} \\ &= \sum_{i=0}^n \sum_{j=0}^n Q_{ij} \delta q_{ij} \end{aligned}$$

Where,

$$Q_{ij} = F \Phi_{i(a/2, b/2)} \Psi_{j(a/2, b/2)} \dots\dots\dots(11)$$

Applying Lagrange's principle to obtain the equations of motion

$$\frac{\partial}{\partial t} \left( \frac{\partial T}{\partial \dot{q}_{rs}} \right) + \frac{\partial V}{\partial q_{rs}} = \{0\} \dots\dots\dots(12)$$

$$\frac{\partial T}{\partial \dot{q}_{rs}} = \frac{m}{2} \sum_{i=0}^n \sum_{j=0}^n \sum_{k=0}^n \sum_{l=0}^n \frac{\dot{q}_{ij}}{\dot{q}_{rs}} \dot{q}_{kl} I_{\Phi_i \Phi_k} I_{\Psi_j \Psi_l} + \frac{m}{2} \sum_{i=0}^n \sum_{j=0}^n \sum_{k=0}^n \sum_{l=0}^n \frac{\dot{q}_{kl}}{\dot{q}_{rs}} \dot{q}_{ij} I_{\Phi_i \Phi_k} I_{\Psi_j \Psi_l}$$

$$= \frac{m}{2} \sum_{i=0}^n \sum_{j=0}^n \sum_{k=0}^n \sum_{l=0}^n \delta_{ir} \delta_{js} \dot{q}_{kl} I_{\Phi_i \Phi_k} I_{\Psi_j \Psi_l} + \frac{m}{2} \sum_{i=0}^n \sum_{j=0}^n \sum_{k=0}^n \sum_{l=0}^n \delta_{kr} \delta_{ls} \dot{q}_{ij} I_{\Phi_i \Phi_k} I_{\Psi_j \Psi_l}$$

Where  $\delta_{ir}, \delta_{js}$  etc are Kronecker delta functions.

$$\begin{aligned} &= \frac{m}{2} \left[ \sum_{k=0}^n \sum_{l=0}^n \dot{q}_{kl} I_{\Phi_r \Phi_k} I_{\Psi_s \Psi_l} + \sum_{i=0}^n \sum_{j=0}^n \dot{q}_{ij} I_{\Phi_i \Phi_r} I_{\Psi_j \Psi_s} \right] \\ &= \frac{m}{2} \left[ \sum_{i=0}^n \sum_{j=0}^n \dot{q}_{ij} I_{\Phi_r \Phi_i} I_{\Psi_s \Psi_j} + \sum_{i=0}^n \sum_{j=0}^n \dot{q}_{ij} I_{\Phi_i \Phi_r} I_{\Psi_j \Psi_s} \right] \\ &= m \sum_{i=0}^n \sum_{j=0}^n \dot{q}_{ij} I_{\Phi_r \Phi_i} I_{\Psi_s \Psi_j} \\ \frac{\partial}{\partial t} \left( \frac{\partial T}{\partial \dot{q}_{rs}} \right) &= m \sum_{i=1}^n \sum_{j=1}^n \ddot{q}_{ij} I_{\Phi_r \Phi_i} I_{\Psi_s \Psi_j} \\ &= [M] \{ \ddot{q} \} \end{aligned}$$

Where  $[M]$  is the mass matrix of order  $n^2 \times n^2$  and elements are given by

$$M_{AB} = m I_{\Phi_r \Phi_i} I_{\Psi_s \Psi_j}$$

Where A and B are the row and column numbers given by

$$A = (r-1)n + s$$

$$B = (i-1)n + j$$

$\{ \ddot{q} \}$  is a vector of order  $n^2 \times 1$

$$\{ \ddot{q} \}^T = \{ \ddot{q}_{11}, \ddot{q}_{12}, \dots, \ddot{q}_{1n}, \ddot{q}_{21}, \dots, \ddot{q}_{nn} \}$$

Similarly the potential energy term differentiated with respect to  $q_r$ , in the Lagrange's equation will give

$$\begin{aligned}
\frac{\partial V}{\partial q_{rs}} &= \sum_{i=1}^n \sum_{j=1}^n q_{ij} \left[ D_{11} I_{\Phi_r'' \Phi_i''} I_{\Psi_s \Psi_j} + D_{12} I_{\Phi_r'' \Phi_i} I_{\Psi_s \Psi_j''} + D_{12} I_{\Phi_i'' \Phi_r} I_{\Psi_s'' \Psi_j} \right] \\
&+ \sum_{i=1}^n \sum_{j=1}^n q_{ij} \left[ 4D_{16} I_{\Phi_i' \Phi_r'} I_{\Psi_j'' \Psi_s''} + D_{22} I_{\Phi_r \Phi_i} I_{\Psi_s'' \Psi_j''} \right] \\
&= [K] \{q\}
\end{aligned}$$

Where  $[K]$  is the stiffness matrix and it's elements are given by

$$\begin{aligned}
K_{AB} &= D_{11} I_{\Phi_r'' \Phi_i''} I_{\Psi_s \Psi_j} + D_{12} I_{\Phi_r'' \Phi_i} I_{\Psi_s \Psi_j''} + D_{12} I_{\Phi_i'' \Phi_r} I_{\Psi_s'' \Psi_j} \\
&+ 4D_{16} I_{\Phi_i' \Phi_r'} I_{\Psi_j'' \Psi_s''} + D_{22} I_{\Phi_r \Phi_i} I_{\Psi_s'' \Psi_j''}
\end{aligned}$$

substituting in terms of matrices will give the equation of a motion as

$$[M] \{\ddot{q}\} + [K] \{q\} = \{0\}$$

Natural frequencies of the plate can be calculated from this equation by equating the forcing function to zero.

Where  $\{0\}$  is a zero vector of order  $n^2 \times 1$

Substituting

$$\{q\} = \{A\} \sin(\omega t)$$

It will give ,

$$-\omega^2 [M] \{A\} + [K] \{A\} = \{0\}$$

$$(\omega^2) = \begin{pmatrix} \omega_{11}^2 \\ \omega_{12}^2 \\ \vdots \\ \omega_{1n}^2 \\ \omega_{21}^2 \\ \vdots \\ \omega_{2n}^2 \\ \vdots \\ \vdots \\ \omega_{nn}^2 \end{pmatrix}$$

$$[K] \{A\} = \omega^2 [M] \{A\} \dots\dots\dots(13)$$

$$\begin{aligned} [[M]^{-1}[K] - \omega^2[I]] \{A\} &= \{0\} \\ [[D] - \omega^2[I]] \{A\} &= \{0\} \dots\dots\dots(14) \end{aligned}$$

Where

$[D] = [M]^{-1}[K]$  is the dynamic matrix

We now define a column vector

Which is the eigen values vector of the equation(14).

Corresponding to each of the eigen values there will be an eigen vector known as shape vector. In order to decouple the equations of motion, the eigen vectors must be normalized. The normalizing procedure, which has been used here involves adjusting each modal amplitude to the amplitude

$Z_r^{(ij)}$  which satisfies the condition

$$\{Z_r\}^T [M] \{Z\} = 1$$

Where,

$Z_r^{ij} = (1/cr^{ij}) \{A_r^{ij}\}$  and  $\{A_r^{ij}\}$  is the eigen vector corresponding to the eigen value  $(\omega_o^2)$ .  $cr^{ij}$  can be calculated for each eigen vector by making the substitution  $Z_r^{ij} = (1/cr^{ij}) \{A_r^{ij}\}$ . Corresponding to each eigen value of the set

$$\lambda_{ij} = (\lambda_{11}, \lambda_{12}, \dots, \lambda_{1n}, \lambda_{21}, \dots, \lambda_{nn})$$

there will be a normalized eigen vector  $Z_r^{ij}$ . A consequence of this type of normalizing together with the modal orthogonality relationships relative to the mass matrix is that

$$\emptyset^T M \emptyset = I$$

Where,

$\emptyset$  is complete set of normalized mode shape vector

$I$  is an  $n^2 \times n^2$  identity matrix.

Using the normalized vector, the equation

$$[K] \{A\} = \omega_o^2 [M] \{A\}$$

can be written as

$$[K] \{Z_r^{11}\} = \omega_o^2 [M] \{Z_r^{11}\}$$

similar equation holds good for  $\omega_{12}, \omega_{13}, \dots, \omega_{1n}, \omega_{21}, \dots, \omega_{nn}$  with corresponding eigen vectors. In the matrix form, it will be

$$[M] [Z_r^{11} \omega_{11}^2, Z_r^{12} \omega_{12}^2, \dots, Z_r^{1n} \omega_{1n}^2, Z_r^{21} \omega_{21}^2, \dots, Z_r^{nn} \omega_{nn}^2] = [K] [Z_r^{11}, Z_r^{12}, \dots, Z_r^{1n}, Z_r^{21}, \dots, Z_r^{nn}]$$

or

$$[M] [Z] [\Psi_o] = [K] [Z] \dots \dots \dots (15)$$

where,

$$\Psi_o = \text{diag} [\omega_{11}^2, \omega_{12}^2, \dots, \omega_{1n}^2, \omega_{21}^2, \dots, \omega_{nn}^2]$$

premultiplying (15) by  $[Z]^T$  gives

$$[Z]^T [M] [Z] [\Psi_o] = [Z]^T [K] [Z]$$

but

$$[Z]^T [K] [Z] = [I] \dots \dots \dots (16)$$

so

$$[Z]^T [K] [Z] = [\Psi_o] \dots \dots \dots (17)$$

Introducing transformation  $\{q\} = [Z] \{p\}$  in the equation

$$[M] \{\ddot{q}\} + [K] \{q\} = \{Q\}$$

where

$$\{Q\}^T = \{Q_{11}, Q_{12}, \dots, Q_{1n}, Q_{21}, \dots, Q_{nn}\}$$

we have,

$$Q_{ij} = F \Phi_{i(a/2)} \Psi_{j(b/2)}$$

Where,

$$F = F_o \sin \omega_f t$$

Making all substitutions in the equation we get

$$[M] [Z] \{\ddot{p}\} + [K] [Z] \{p\} = \{Q\}$$

Multiplying by  $[Z]^T$

$$[Z]^T [M] [Z] \{\ddot{p}\} + [Z]^T [K] [Z] \{p\} = [Z]^T \{Q\} \dots \dots \dots (18)$$

Substituting equations (15) and (16) in (18)

$$\{\ddot{p}\} + [\Psi_o] \{p\} = [Z]^T \{Q\}$$

Equations are decoupled .The solution is

$$P_{ij} = \frac{1}{(\omega_{ij}^2 - \omega_f^2)} \sum_{k=1}^{n \times n} Z_{ki} \{Q\} \sin \omega_f t$$

Where,

$$l = (i - 1) n + j$$

Substituting back for  $\{q\} = [Z] \{p\}$  ,we can calculate the deflection in terms of generalized coordinates.From this we can calculate the lateral deflection at any point on the plate using equation(1).

Certain idealization made for calculating the crystal response are as follows:

1. It is assumed that the sensors were perfectly bonded to the substructure and the strains in the sensors are compatible with the surrounding structure.
2. The strains developed in the sensors are within the elastic limit.
3. The sensors are idealized to be of point size.

Strains are developed at any point in the plate are given by

$$\epsilon_{xx} = -z \frac{\partial W}{\partial x}$$

$$\epsilon_{yy} = -z \frac{\partial W}{\partial y}$$

Where  $z$  = distance of mid plane of sensor from the neutral axis of plate.

$W$  = Lateral deflection of the plate.

According to the first idealization, strains in the plate are equal to the strains in the crystal at that location.

$$(\epsilon_{xx})_p = (\epsilon_{xx})_c$$

$$(\epsilon_{yy})_p = (\epsilon_{yy})_c$$

Strains developed by the sensors due to electric energy are

$$(\epsilon_{xx})_c = d_{31}V_x/t_x$$

$$(\epsilon_{yy})_c = d_{31}V_y/t_y$$

and

$$(\epsilon_{zz})_c = d_{33}V_z/t_z = 0$$

Where,

$d_{31}$  and  $d_{33}$  are piezoelectric constants

$(\epsilon_{zz})_c$  is zero because the normal stresses in the thickness direction of the plate

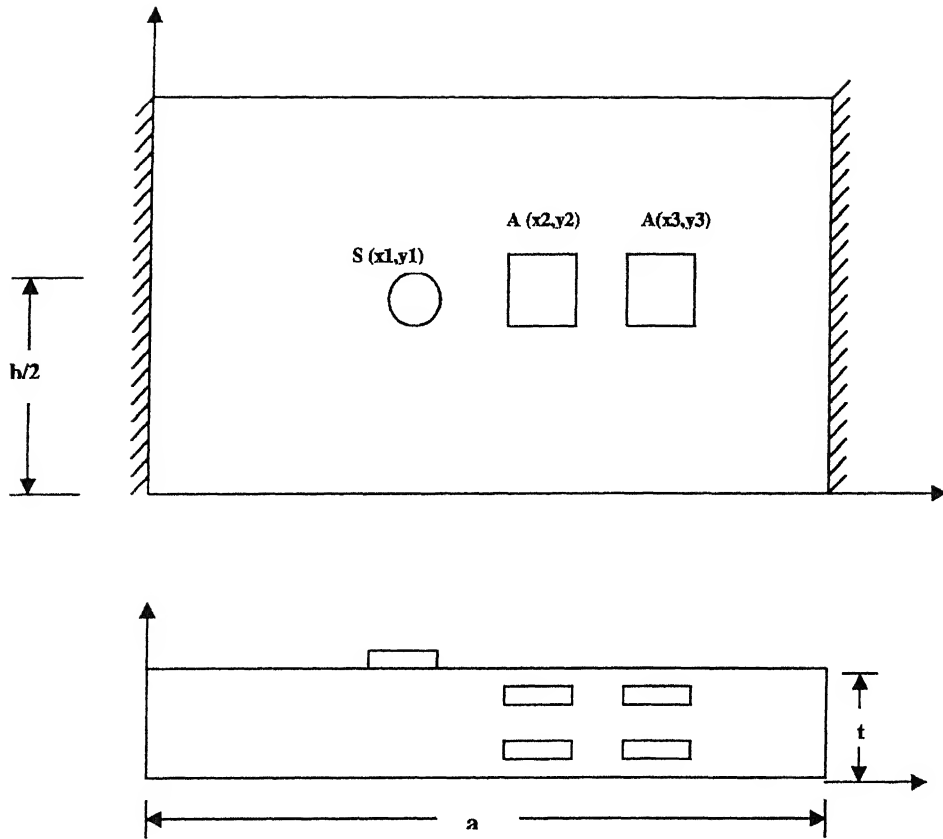


are assumed to be zero.

$V_x$  and  $V_y$  =voltage required to induce strains in the actuators.

$t_x$  and  $t_y$  = thickness of the actuator in x and y direction.

Therefore knowing the strains developed in the actuators are same as that the plate,we can calculate the voltages required to generate the same amount of strains.



Where ,

$a$  =Length of the plate.

$b$ =Width of the plate.

$t$  =Thickness of the plate.

$S_1$ =Accelerometer located at  $(x_1, y_1)$

$S_2$ = Sensor located at  $(x_2, y_2)$  &

$A$ = Actuator located at  $(x_3, y_3)$

Figure-6:Clamped plate used for the dynamic test

## 4.2 EXPERIMENTAL PROCEDURE FOR VIBRATION CONTROL

### 4.2.1 Manufacturing of glass/epoxy specimen

The smart glass/epoxy test plate was manufactured using glass/epoxy prepregs and embedded with piezoelectric crystals. The dimensions of the test specimen and the crystal details are given in Table-5. Two crystals were embedded inside the plate.

### 4.2.2 Embedding of piezoelectric crystals

Leads of diameter 0.2 mm were flattened and attached to crystals by soldering from top and bottom to each of the crystals. The overall thickness of crystals along with leads is about 0.35 mm. Thickness of each prepreg is 0.119 mm. So to accommodate the two crystals in the laminate, fibers from 3 layers at equal distance from neutral axis of the plate were cut out by using a surgical blade. The epoxy used between the layers is a mixer of Araldite and Hardener in the ratio of 1:10. Crystals were placed in the respective holes while stacking the prepregs. Finally the stacked prepregs were cured for 24 Hours to form a laminate.

Structure	Material	Dimensions (mm)
1. Plate	Glass fabric/epoxy	260×160×3.4
2. Actuator	PZT	12×12×0.25
3. Sensor	PZT	ϕ10

Table-5: Plate and crystal details.

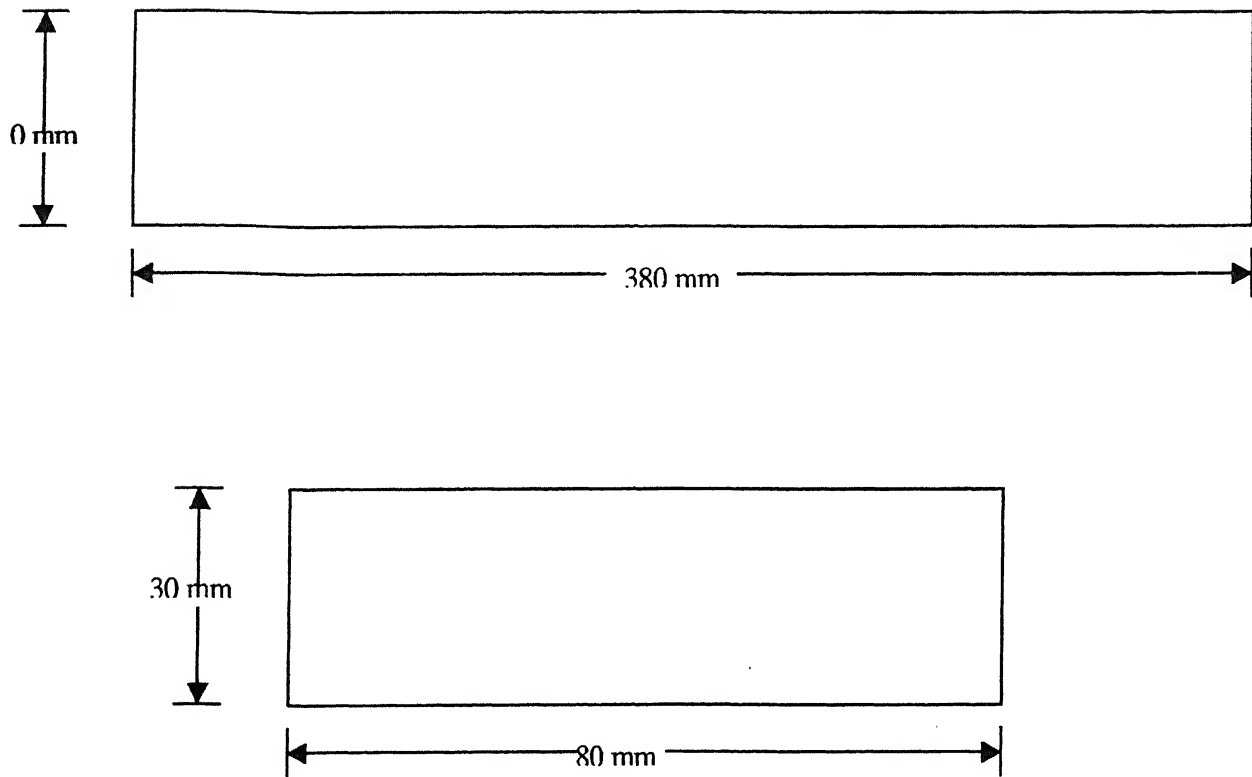
### 4.2.3 Bonding of end tabs

Aluminium end tabs cut from an aluminium sheet with shearing machine. The dimensions of the end tabs are given in figure-7. End tabs were bonded on the top and bottom of the two ends of the plate to avoid delamination of plate due to clamping. The bonding agent was epoxy.

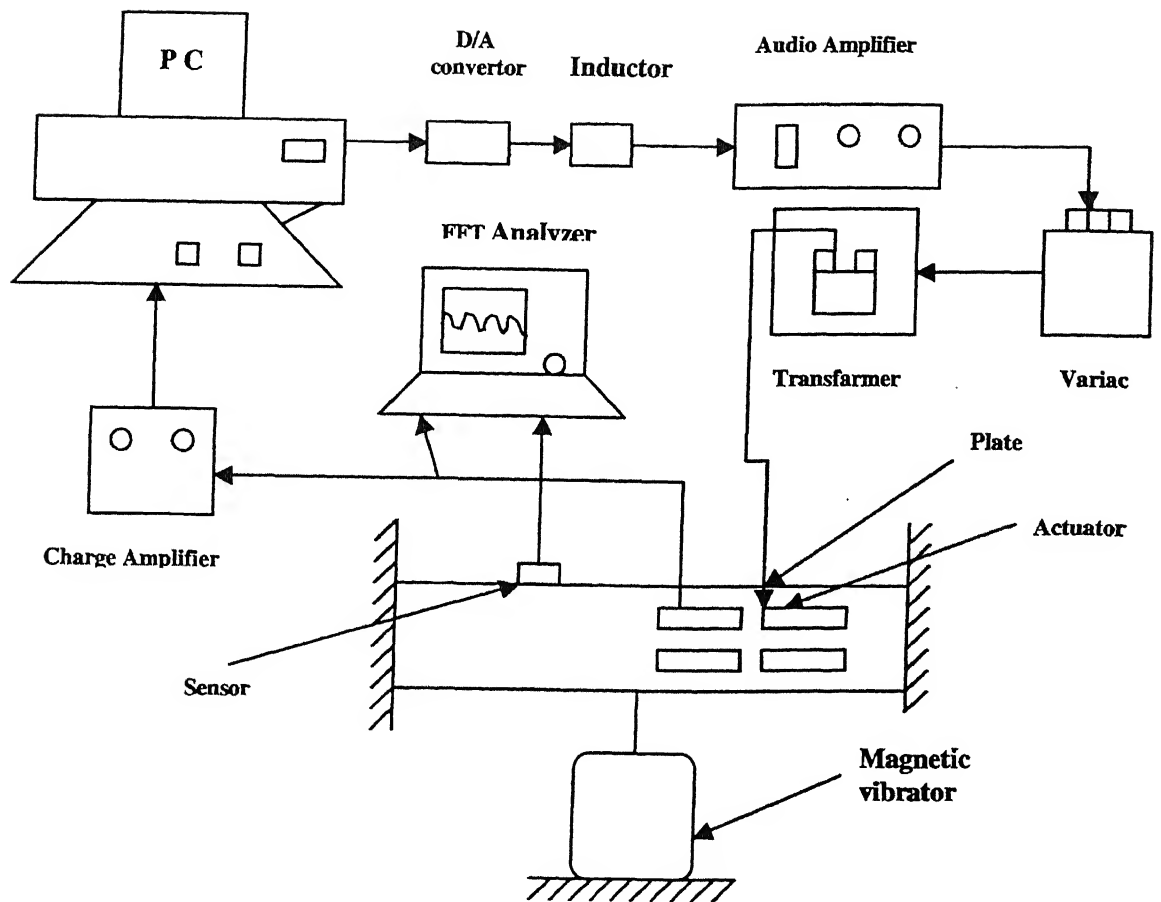
## Experimental procedure for vibration control

For clamping the plate , four 'C'- clamps were used , two at one end and two at the other. The plate was clamped to a heavy iron frame structure. The plate was vibrated by using the external magnetic vibrator and one pair of crystals were being used as sensors to take the vibration signals which were embedded in the plate. These signals were being taken into the PC to have the information about the signals before and after the vibration control. The output of PC was passed through the inductor and RC circuit to make the signals  $180^\circ$  out of phase. Afterwards these inverted signals were amplified through amplifier and step-up through transformer of 1:4 ratio. This amplified output was then supplied to the next pair of crystals to control the modes vibration. Vibration responses were taken by using the FFT analyzer and PC used to take the data points on online to compare the frequency responses before and after vibration control.

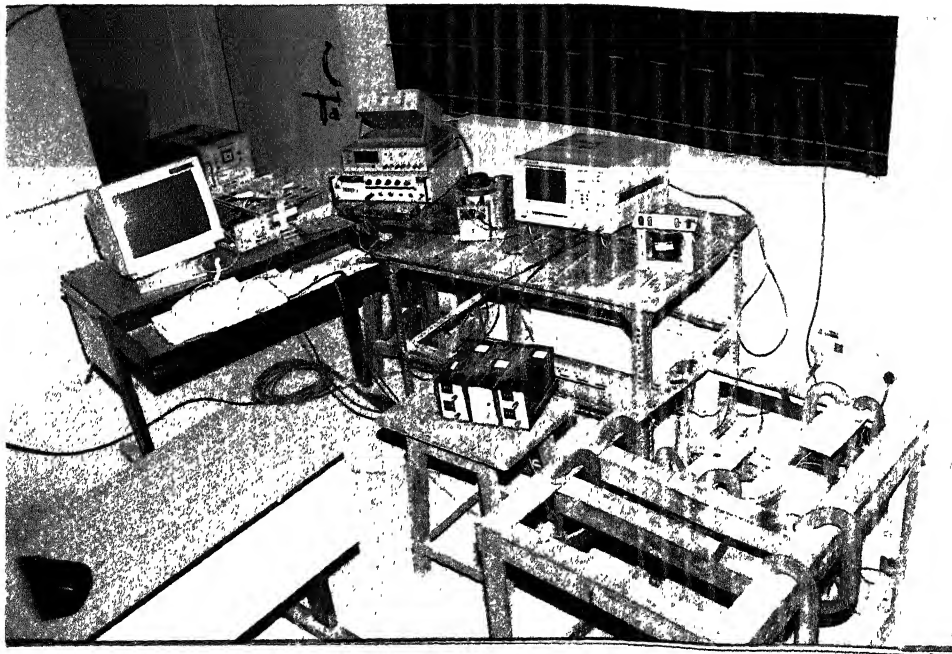
The block diagram of the experimental set up, photo of clamped-clamped and cantilever plate are shown in figure-8 & 9 respectively. Experimentally free vibration testing was done by tapping the plate with hand. This was done to know the first three natural frequencies of the plate for the given boundary conditions.



**Figure-7:End tabs for the composite plate**



**Figure-8:Block diagram of experimental set up for clamped-clamped plate with feedback control system.**



**Figure-9: Schematic diagram of experimental set-up for Cantilever plate with feedback control system.**

# Chapter 5

## RESULTS AND DISCUSSION

### 5.1 Closed loop feedback controller

Vibration control of the Cantilever plate was successfully done by using closed loop feedback controller with proportional control action. Feedback control system consists of the following elements:

- 1.Charge Amplifier
- 2.PC with PCL-812 Card
- 3.RC circuit , inductor
- 4.Amplifier
- 5.Transformer

Voltage generated by the sensor was successfully amplified to the required voltage to control the vibration of the plate. In the feedback control system ,PC was introduced to have the data points of the signals coming from sensors to compare the frequency responses before and after vibration control. Data was processed through code by using PCL-812 data acquisition card. Figure-10 shows the signal flow chart for PC.

### 5.2 Free vibration responses of plates

Experimental set up for free vibration responses is shown in figure-11. The clamped-clamped plate was tapped and responses were being taken on the FFT analyzer. Thus experimentally we got first three natural frequencies. Also natural frequencies were being calculated from theoretical formulation through code. Comparison between experimental and theoretical natural frequencies are given in table-6.



Mode	Experimental values (Hz)	Theoretical values (Hz)
1	165	164
2	325	345
3	475	478

**Table-6:Comparison of theoretical and experimental natural frequencies of clamped-clamped plate**

Experimental set up for cantilever plate was same as for clamped-clamped plate. Cantilever plate was tapped and responses were being recorded on FFT analyzer. The time response and frequency response of cantilever plate are shown in figure-12 &13. The first three natural frequencies were found out experimentally are shown in table-7.

Mode	Experimental values (Hz)
1	155
2	465
3	895

**Table-7:Experimental natural frequencies of cantilever plate**

### 5.3 Dynamic responses of plates

Sinusoidal excitation of constant frequency 165 Hz was applied to the magnetic shaker to excite the plate. The acceleration of the plate experimentally at this frequency was calculated from the amplitude of the charge generated by the sensors. Using acceleration and plate properties ,excitation force on the plate as well as deflection were calculated are given in Table-8.

Experimental Value of Acceleration of the clamped-clamped plate found out is  $59.1757m/s^2$ .

1	Excitation force	87.853 N
2	Deflection of the plate	0.7 mm

**Table-8:Excitation force and deflection of clamped-clamped plate at 165 Hz frequency.**

**Vibration responses of clamped-clamped plate by using feedback control system:**

The clamped-clamped plate was excited by the shaker at first natural frequency and the responses were recorded before and after vibration control shown in figure-14&15. Before performing the vibration control test on the plate, maximum voltage (208V) was supplied to the actuators through open loop system. It was found that the force generated by the actuator was not even sufficient to vibrate the clamped-clamped plate. So we removed one fixed boundary condition of the plate and actuators were supplied with the same voltage. It was found that the cantilever plate was vibrating. So we perform the experiments on the cantilever plate for vibration control by using feedback control system as showed in figure-9.

**Vibration responses of cantilever plate by using feedback control system:**

The first three natural frequencies of the cantilever plate are already given in table-7. Hence sinusoidal excitation of constant frequencies 155 Hz ,465 Hz and 895 Hz were applied one by one to the magnetic shaker to excite the plate. Vibration control for cantilever plate was successfully done by using feedback control system with proportional controller. Dynamic responses before and after the vibration control are being shown in figure-16,17,18,19,20,21,22,23,24,25,26 & 27 respectively.

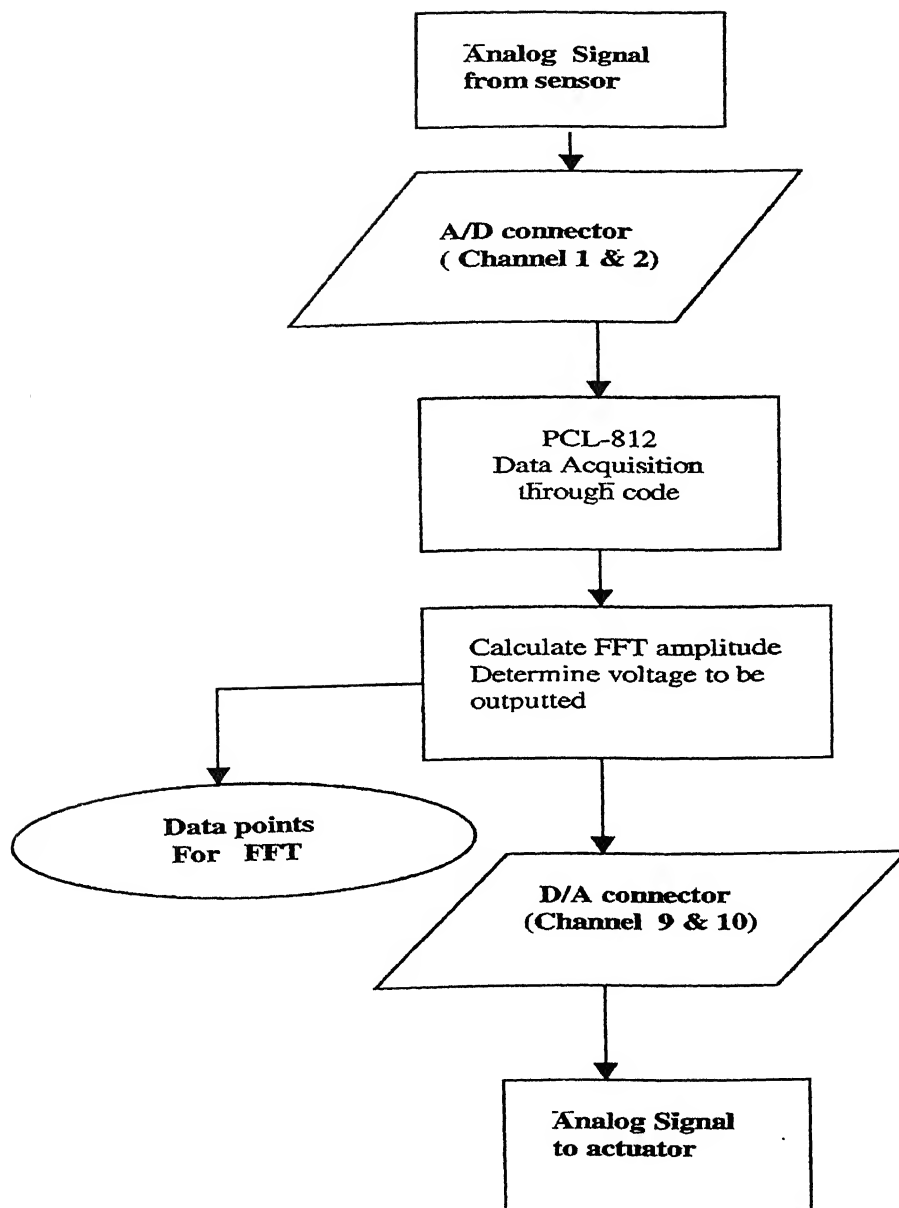
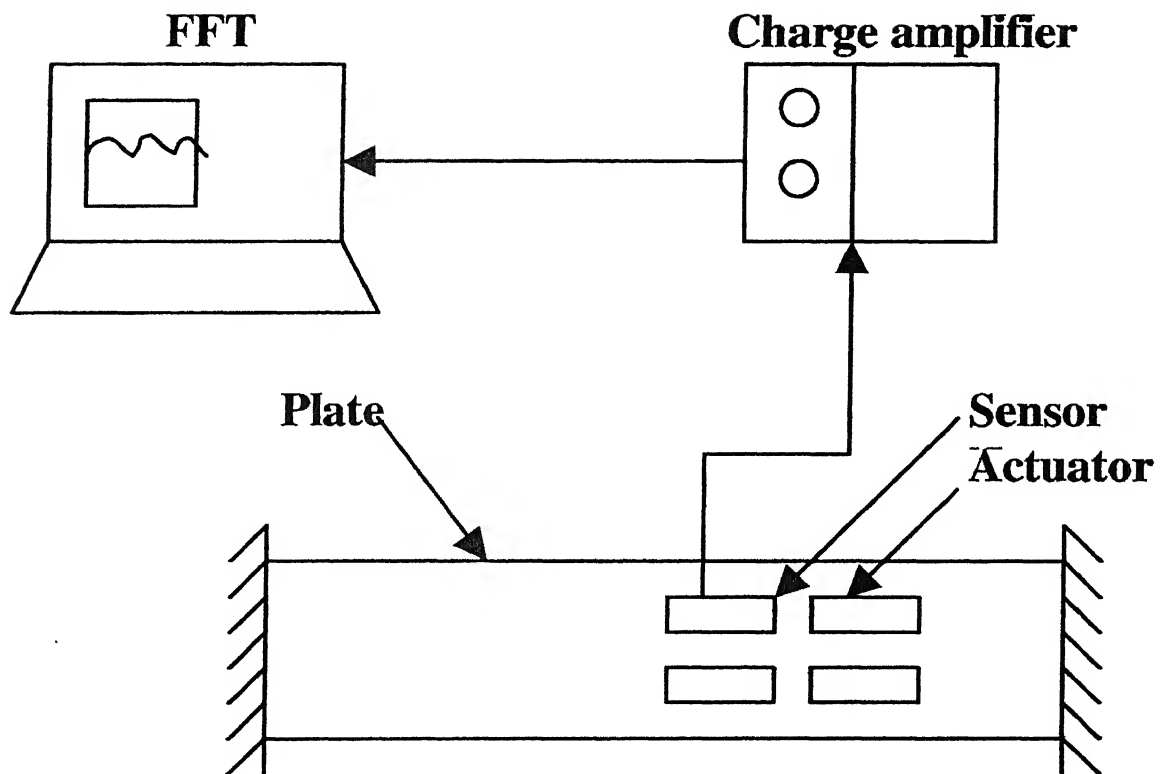


Figure-10:Signal flow chart for PC



**Figure-11:Experimental set up for free vibration response of clamped-clamped plate**

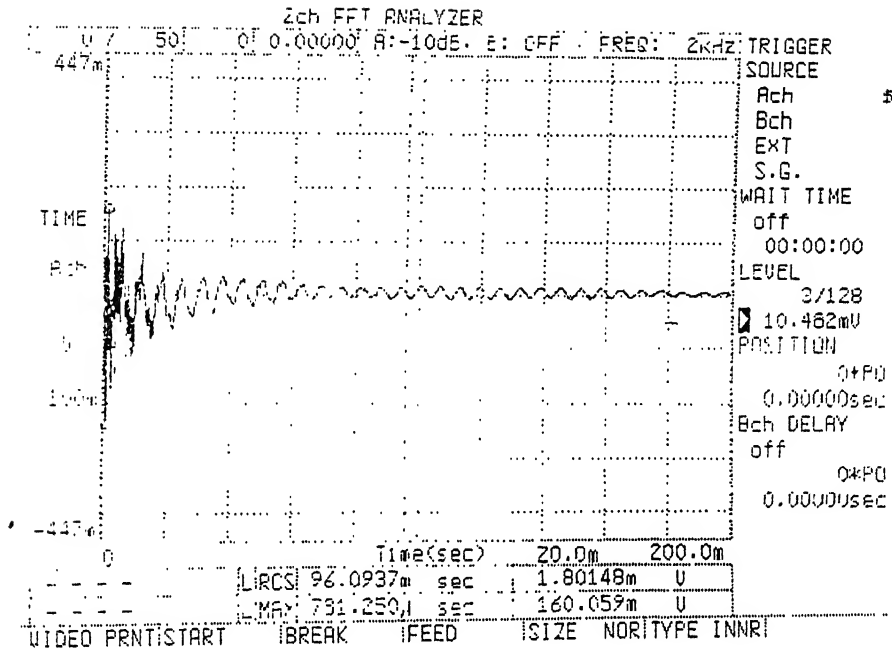


Figure-12: Time response of cantilever plate after tapping

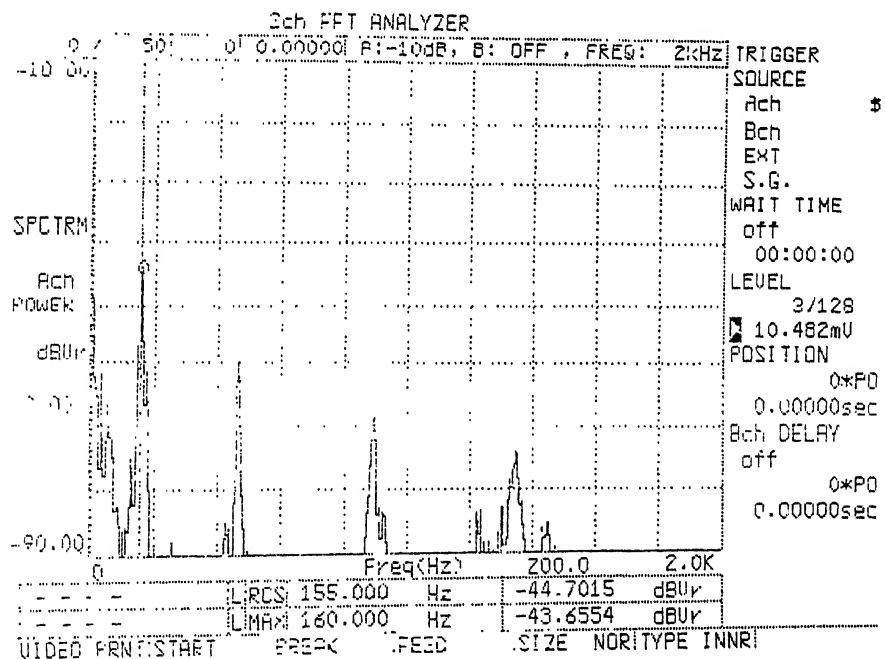
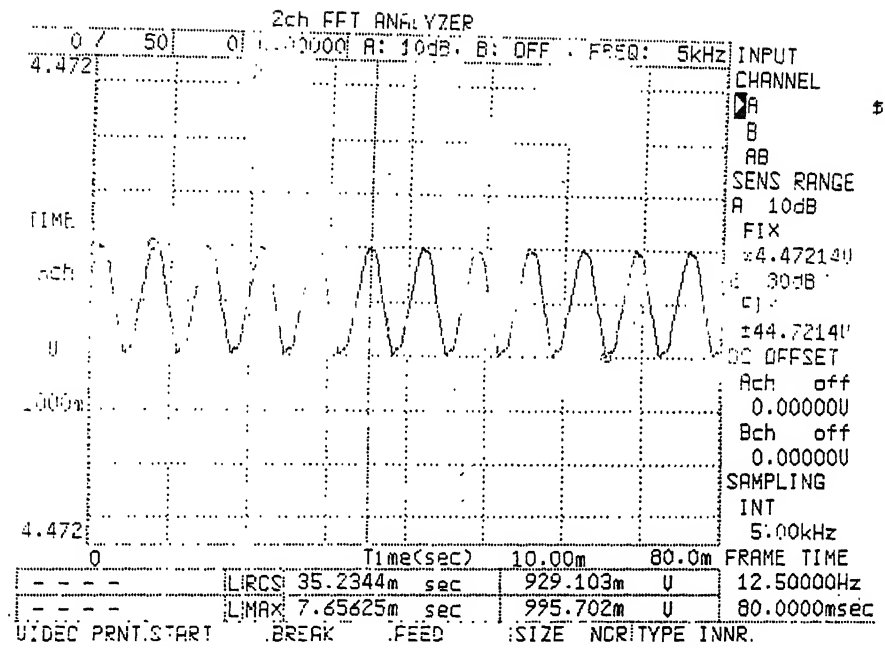
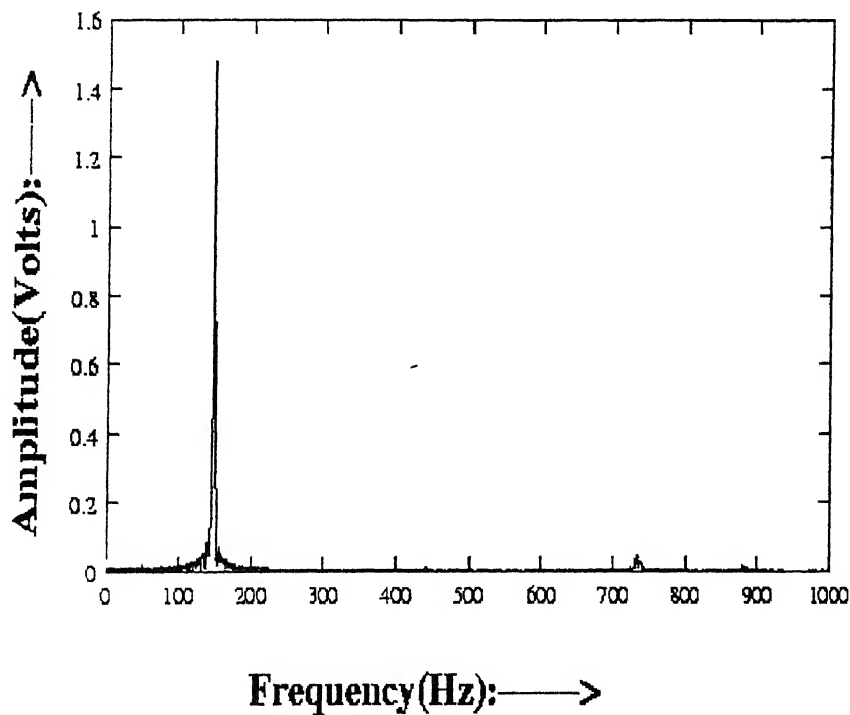


Figure-13: Frequency response of cantilever plate after tapping



**Figure-14:Time response of clamped-clamped plate before and after vibration control at 165 Hz**



**Figure-15:Frequency response of clamped-clamped plate before and after vibration control at 165 Hz**

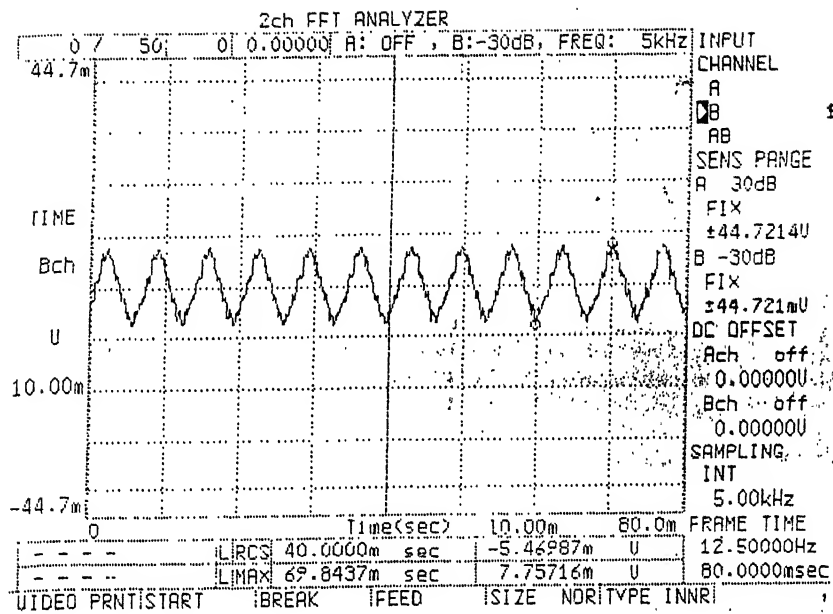


Figure-16: Time response of cantilever plate before vibration control at 155 Hz

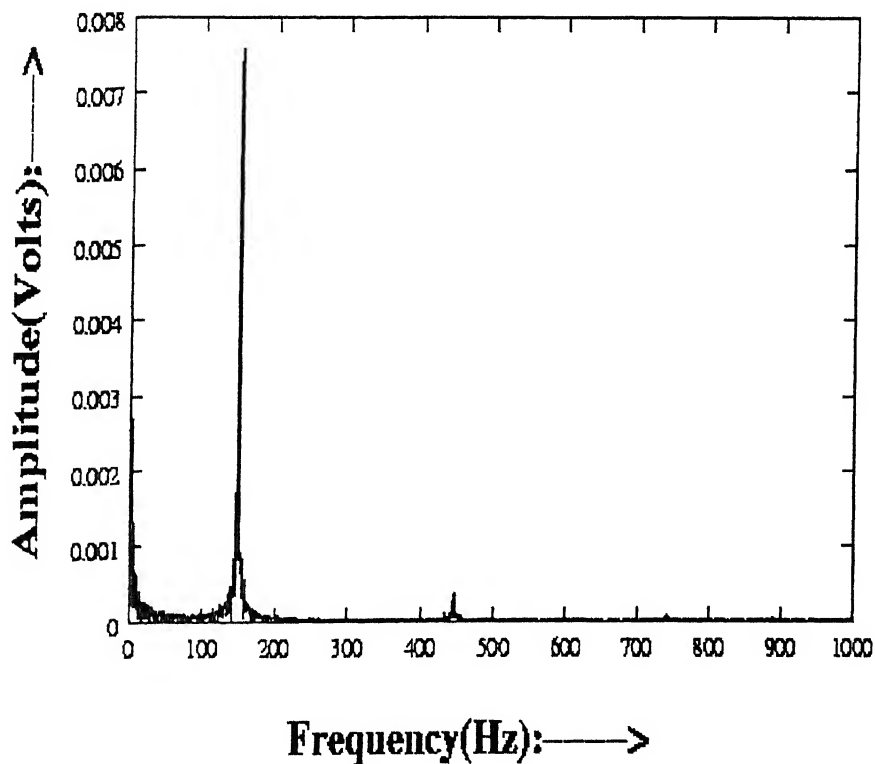


Figure-17: Frequency response of Cantilever plate before vibration control at 155 Hz .

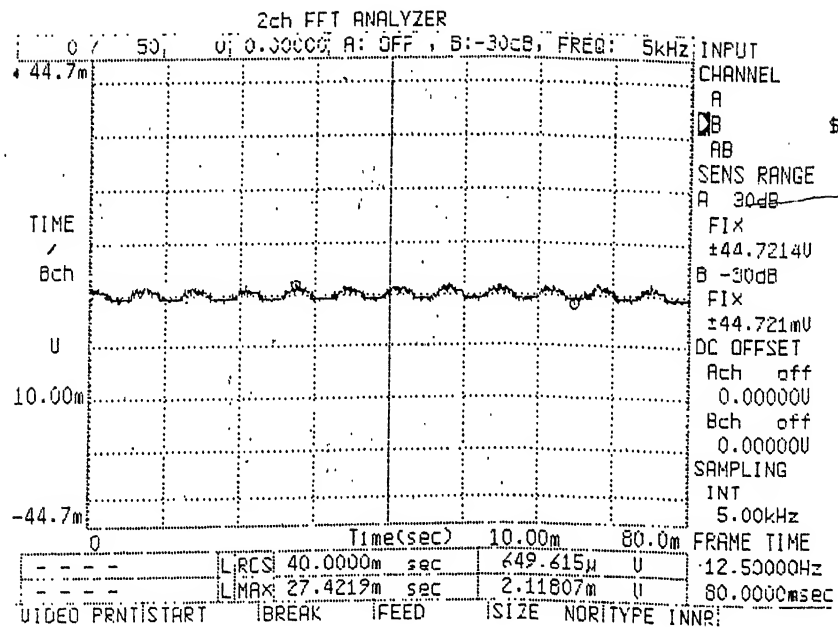


Figure-18: Time response of cantilever plate after vibration control at 155 Hz

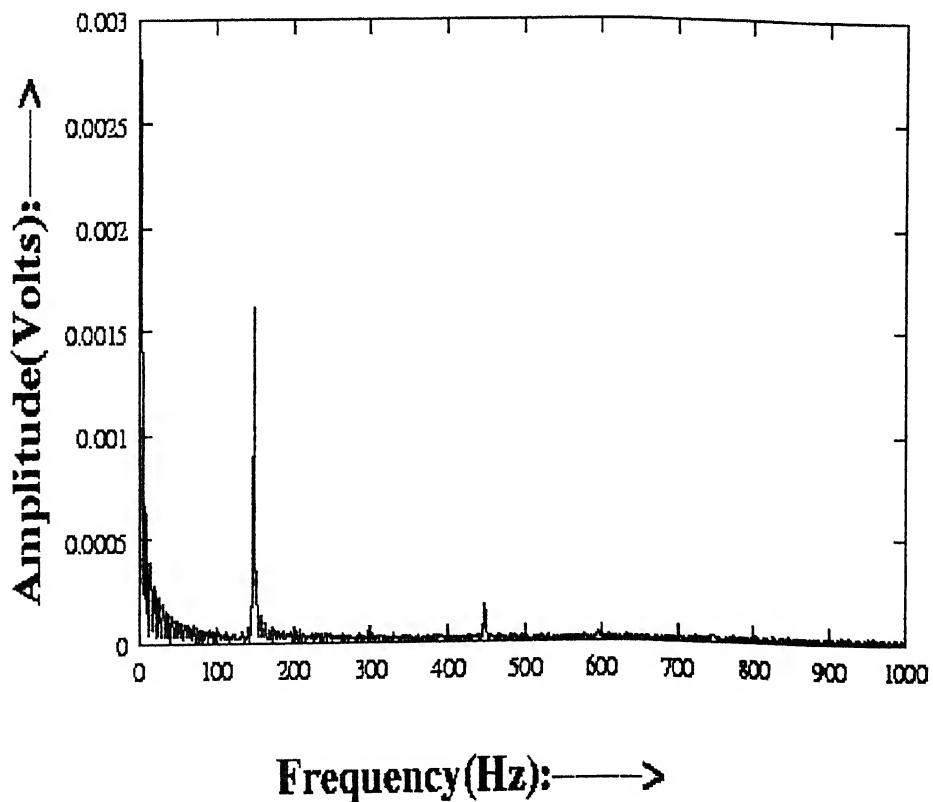


Figure-19: Frequency response of cantilever plate after vibration control at 155 Hz.



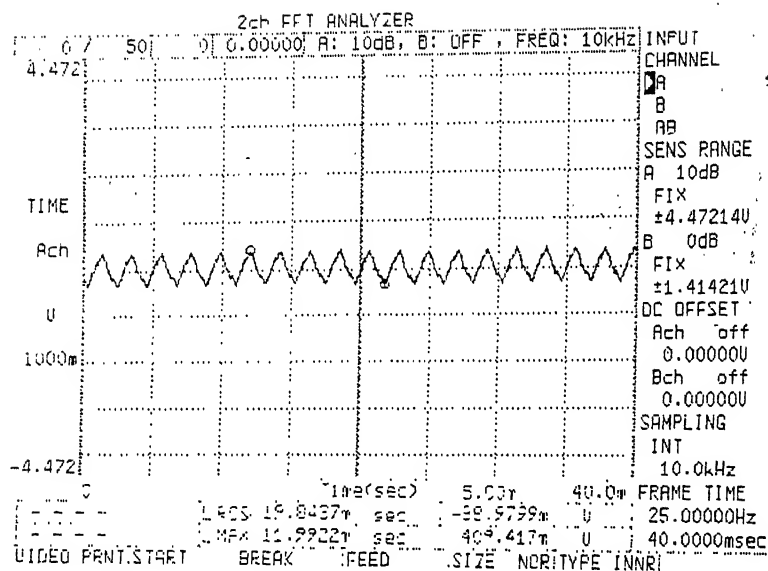


Figure-20: Time response of cantilever plate before vibration control at 465 Hz

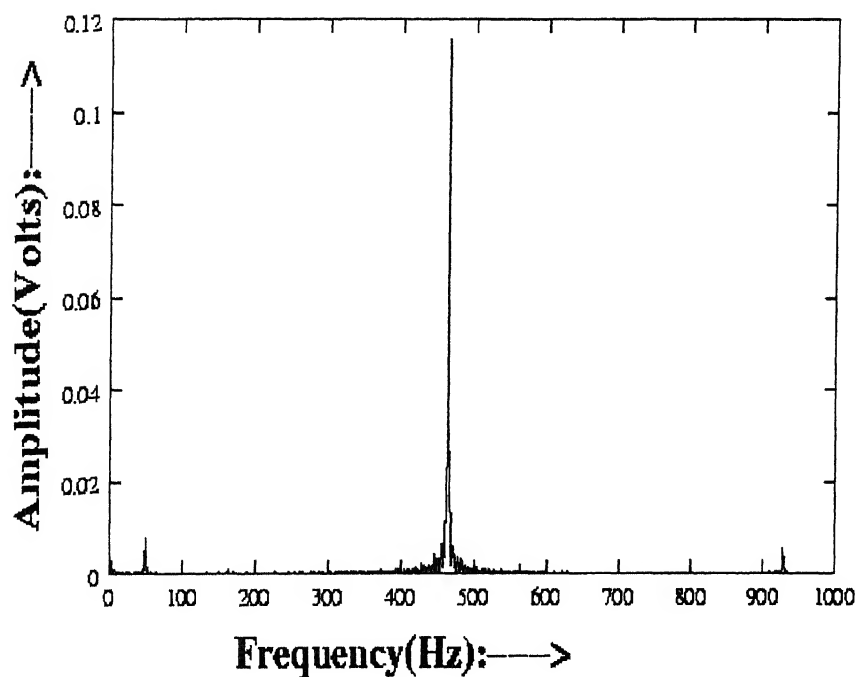
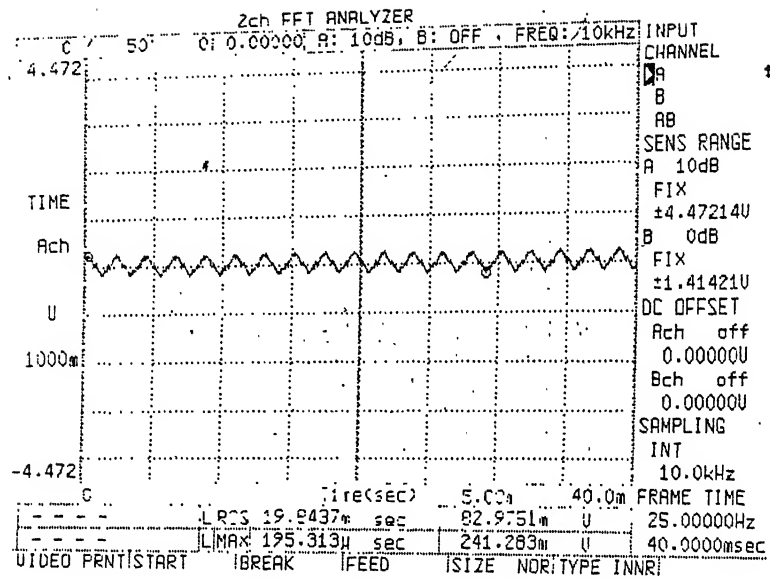
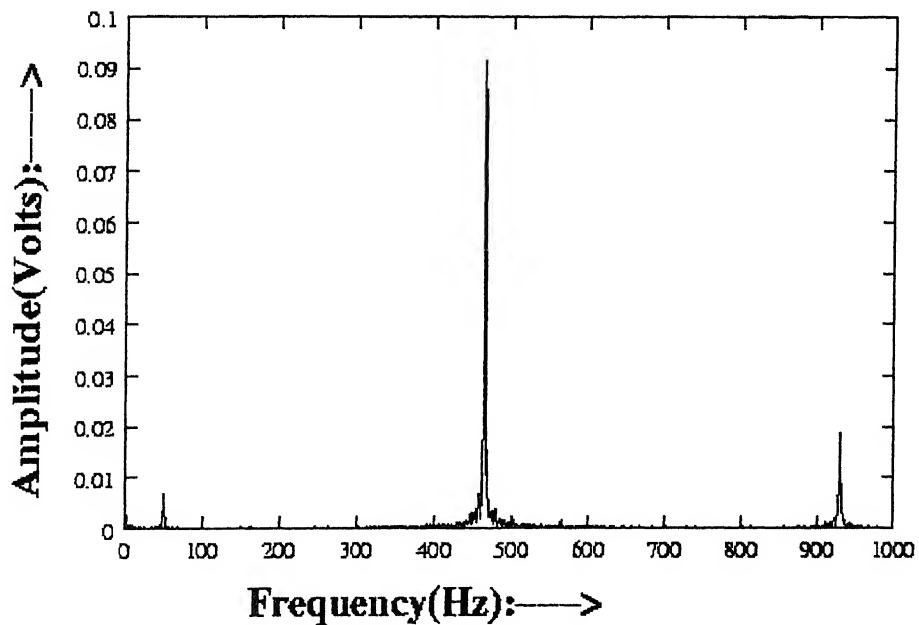


Figure-21: Frequency response of cantilever plate before vibration control at 465 Hz.



**Figure-22:Time response of cantilever plate after vibration control at 465 Hz**



**Figure-23:Frequency response of cantilever plate after vibration control at 465 Hz.**



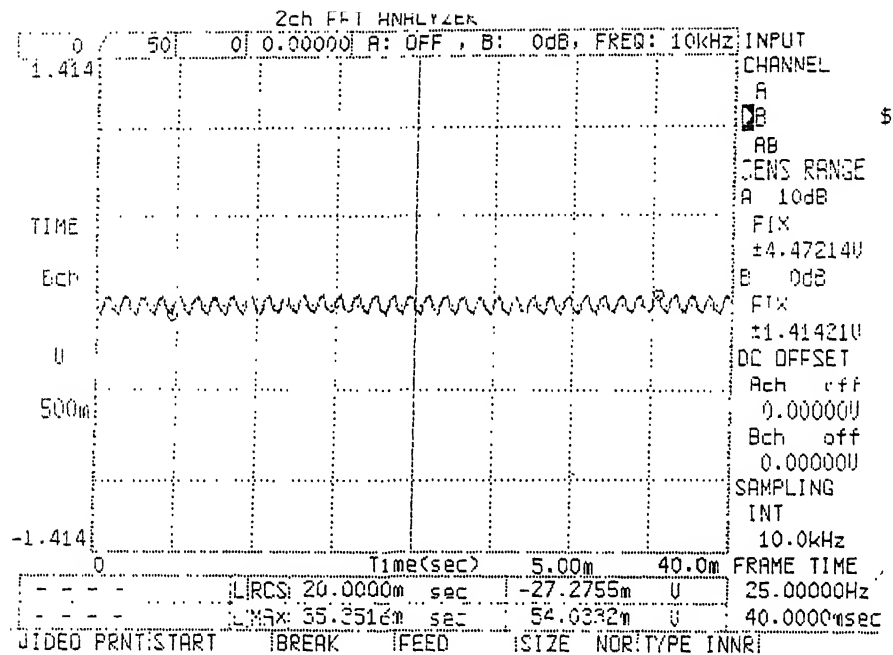


Figure-26: Time response of cantilever plate after vibration control at 895 Hz

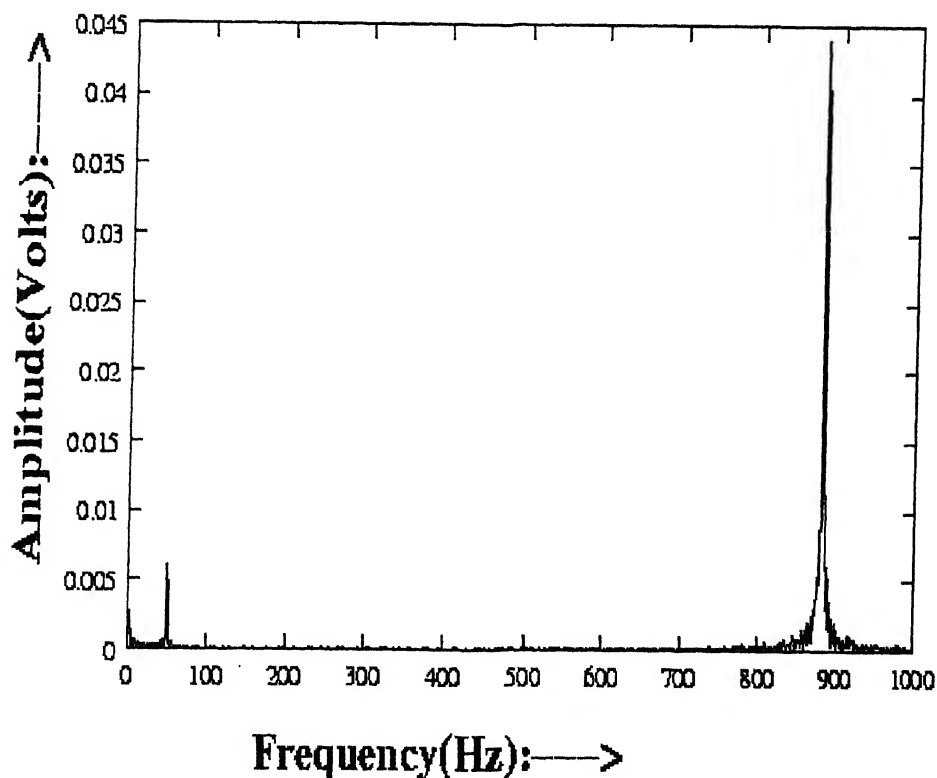


Figure-27: Frequency response of cantilever plate after vibration control at 895 Hz.

# Chapter 6

## CONCLUSIONS AND FUTURE WORK

The present work deals with the vibration control of a composite plate by using feedback control system with proportional control action, where PZT crystals are used as both sensor as well as actuator.

The natural frequencies of a composite plate embedded with piezoelectric crystals were calculated by tapping the plate with hand. The responses of the crystals were conditioned with a charge amplifier and analyzed with the help of a FFT analyzer. In the second experiment, output from the sensors was employed to control the plate vibration by using the feedback control system. Analog signals generated by the sensors/charge amplifier were inputted into the PC through an A/D convertor. The FFT of the input data was calculated and used to determine the requisite output voltage. The voltage was outputted through a D/A convertor port and the DC components of the output signals were removed by using a RC circuit. The output signals were made  $180^\circ$  out of phase by using an inverter/circuit and stepped up through a transformer.

The aim of this work is to control the first three modes of vibration of the plate by using the feedback control system. It was successfully done to control the vibrations of cantilever plate by using the feedback control system with proportional control action. Salient features of the present work are:

1. Data processing by using PCL-812 data acquisition card.
2. Natural frequencies of the clamped-clamped plate were calculated theoretically and compared with the experimental values.
3. Dynamic responses of the clamped-clamped and cantilever plate were before vibration control compared with after vibration control.
4. Vibration control of the cantilever plate was successfully done by using feedback control system with proportional control action.

## Future work

1. With experience gained in this work , one can address the same structure with various boundary conditions.
2. One can go for complex geometries like shells, trusses etc for vibration control.
3. Optimization of the locations of actuators and sensors as well.
4. One can go for good signal conditioning circuits for better performance of the feedback control system.

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